

# **RESPONSE OF A CANTILEVER COLUMN UNDER FOLLOWER LOADS**

**By**

**VISHNU KUMAR**



**DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR  
DECEMBER, 1979**

# **RESPONSE OF A CANTILEVER COLUMN UNDER FOLLOWER LOADS**

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**By**

**VISHNU KUMAR**

**to the**

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DEDICATION TO  
MY PARENTS

## CERTIFICATE

This is to certify that the work presented in  
this thesis entitled "Response of Cantilever Column Under  
Follower Loads" by Shri Vishnu Kumar has been carried out  
under my supervision and it has not been submitted  
elsewhere for a degree.



V. Sundararajan  
Professor

Department of Mechanical Engineering  
Indian Institute of Technology Kanpur

December 1979

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## LIST OF SYMBOLS

A	Cross-sectional area of the pipe
c	Dimensionless parameter for external damping
E	Modulus of elasticity
$E^*$	Coefficient of internal dissipation
EI	Flexural rigidity
K	Coefficient of external dissipation
L	Length of the pipe
M	Magnification factor
$M^*$	Maximum magnification factor
$m_f$	Mass of fluid/unit length
$m_p$	Mass of pipe/unit length
$P_o$	External follower load
$P(\tau)$	$= P_o L^2/EI$
$P_{cr}$	Critical follower load
q	Function of dimensionless time
r	Mass ratio ( $\frac{\text{mass of fluid}}{\text{mass of pipe} + \text{mass of fluid}}$ )
t	Time
U	Mean flow velocity
u	Dimensionless parameter for flow velocity
v	Instantaneous velocity of a fluid element
w	Dimensionless parameter for transverse deflection of pipe
x	X - axis (coincides with the centre line of undeflected pipe )
y	Transverse deflection of pipe

$$\alpha_m = \frac{\cos(\beta_m) + \cosh(\beta_m)}{\sin(\beta_m) + \sinh(\beta_m)}$$

$\beta_m$  Roots of frequency equation

$\gamma$  Dimensionless parameter for internal damping

$\xi$  Dimensionless parameter for  $x$

$\tau$  Dimensionless parameter for time

$\tau^*$  Load duration for  $P(\tau)$

$\phi_m$  Orthonormal eigenfunction

$$\phi_{mn} = \int_0^1 \phi_m \frac{d^2 \phi_n}{d\xi^2} d\xi$$

$$\psi_{mn} = \int_0^1 \phi_m \frac{d \phi_n}{d\xi} d\xi$$

$$\omega_m = \beta_m^2$$

## ABSTRACT

The present work deals with the response of a cantilever pipe conveying fluid under transient compressive follower load. The internal damping, external damping fluid flow, coriolis component due to fluid flow and external compressive tangential load at the free end of the pipe are taken into account for the formulation of problem. The partial differential equation describing the motion is reduced to a set of coupled linear ordinary differential equations with the aid of Galerkin's technique and the resulting equations are numerically solved by Haming's method.

The external damping stabilizes whereas the internal damping destabilizes the nonconservative system. The fluid damping and coriolis force due to it, overcome the destabilizing effect of internal damping. Whether or not the maximum response will increase or decrease as the load duration increases seems to depend on the deflection shape of the column at the instant the load is released. For a particular load and load duration the second mode dominates the overall response of the column. This indicates that in the case of very high loads acting for a short duration the second mode response also is of importance. The magnification factor is a good measure of the overall response but does not always correspond to maximum response.

## CHAPTER 1

### INTRODUCTION

The study of the response and stability of structural elements under nonconservative force has become increasingly important due to the application in a variety of fields. After classifying the various types of forces, a brief survey of the existing literature on the stability of nonconservative system is presented. Only those literature which is relevant to the present work is touched upon.

#### 1.1 Classification of Forces

Forces which act on any structure can be classified broadly into active and reactive forces. Active forces can further be classified as shown in Fig. 1

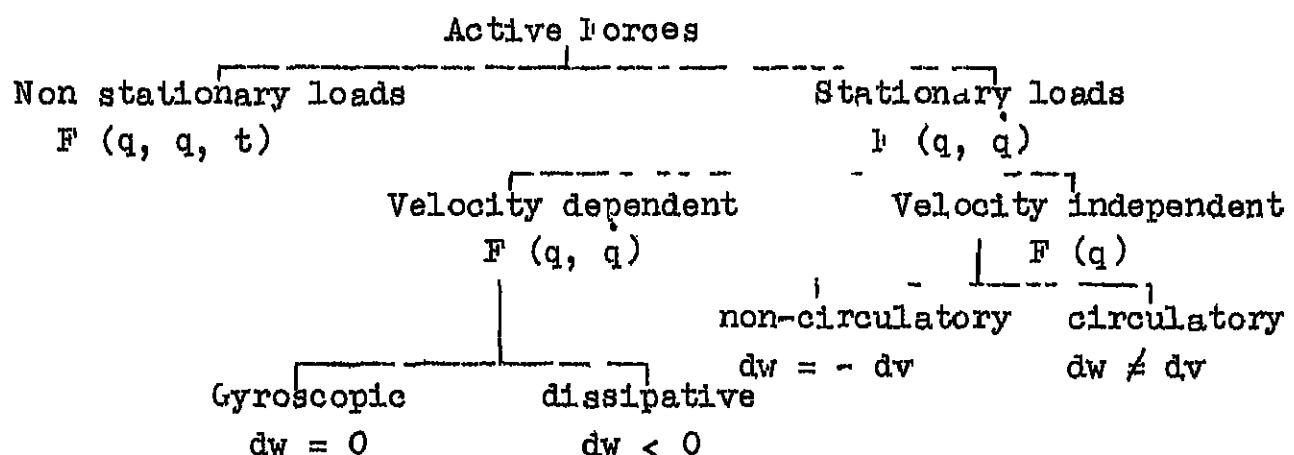


FIGURE 1. CLASSIFICATION OF FORCES

Nonstationary loads depend on time explicitly whereas stationary loads do not. The coriolis force due to fluid flow is the example of gyroscopic force, and material damping, external damping and fluid friction are the example of dissipative forces. Under the category of velocity independent forces we have the noncirculatory and circulatory forces. Constant loads are the example of noncirculatory forces.

This classification enables us to put the stability problems of nonconservative system, in three distinct groups. The first is concerned with elastic systems subjected to the so called follower forces, that is, forces which follow in a prescribed manner the deformation of the system. Such forces called the "circulatory" forces, are nondissipative and in general nonconservative. A second group deals with stability of rotating shafts (whirling) and the third and last group with stability of elastic bodies placed in moving fluid (aeroelasticity). In this work we are concerned with the first type of stability problem.

## 1.2 Destabilizing Effect of Internal Damping

Since the pioneering studies of Ziegler in 1952 on the destabilizing effect of internal damping of non-conservative elastic systems, many researchers have investigated the various aspects of the influence of damping.

Nemat - Nasser and Herrmann [1] , Nemat - Nasser [2] , Prasad and Herrmann [3] and Bolotin and Zhiqinger [4] have shown that the critical load of an undamped discrete or continuous, elastic system under follower forces is an upper bound for the critical load of the system with slight internal damping Jong [20] showed for the case of two degree of freedom systems that the flutter boundary curves of the undamped systems are the envelope of the family of flutter boundary curves of systems with small internal damping The destabilizing effect was further elaborated by Bolotin [5] , who considered a general system with two degree of freedom not related to any particular mechanical model and who found, additionally, that the destabilizing effect in the presence of slight and vanishing damping is highly dependent on the relative magnitude of damping coefficients in the two degrees of freedom

Additional insight into destabilizing effects of linear velocity dependent damping in nonconservative system was supplied by Herrmann and Jong [6] They studied the roots of the characteristic equation, in addition to the stability criteria and introduced the concept of degree of instability. They established a relationship between critical loading for no damping, for slight damping and for vanishing damping It was found that while the presence of small damping may have a destabilizing effect, proper interpretation of the limiting process of

vanishing damping leads to the same critical load as for no damping. Herrmann and Long [7] showed further that for certain systems instability by divergence or by flutter may occur depending upon the ratio of the damping coefficients in the degrees of freedom.

However, the destabilizing effect of viscous damping in a linear, dynamic system with two degrees of freedom, subjected to nonconservative forces has not been put in a strong theoretical foundation. Nor it has been shown whether a more general system with many degrees of freedom can also exhibit such behaviour. Moreover from examples worked out in [5, 6, 7] it is not clear whether other velocity dependent forces [8, 9] can have similar effects.

Nemat - Nasser and Herrmann[10] have proved that in a general nonconservative system with  $N =$  degrees of freedom not only slight damping but all sufficiently small velocity dependent forces may have a destabilizing effect.

This effect in continuous systems was considered by Nemat - Nasser, Prasad and Herrmann [11]. The velocity dependent forces may be due to either internal or external damping, or to coriolis forces (in pipes conveying fluid) or to other gyroscopic forces. For continuous systems with slight damping, Nemat - Nasser [12] proved that the flutter load parameter of the undamped system is an upper bound for that system with slight damping.

### 1 3 Effect of External Damping

The effect of external damping on a linear elastic cantilevered column subjected to a constant follower force at its free end, is not destabilizing, has been shown by Nemat-Nasser, Prasad and Herrmann [11]. It is shown by Plaut and Infante [12] that the critical load increases with increasing damping, from the value of  $20.05 \frac{EI}{L^2}$  at zero damping to the limiting value of  $37.7 \frac{EI}{L^2}$  for large damping. This behaviour is in marked contrast to that of the internally damped column and also to that of the conservative systems (where external damping has no effect on the critical load).

### 1 4 Stability of Pipes Containing Flowing Fluid

Benzamin [13] has investigated the, when the tube is fixed at one end and free at the other, large flow velocities can cause instability in certain modes of vibration, whereas for smaller velocities all possible modes are damped.

How assumptions were given by Benzamin for the formulation of problem of solid and fluid interaction. If the lengths of the pipes are much larger than the dimensions of their cross-sections, it is reasonable to assume that the small scale details of flow, for example, turbulence and secondary flows arising at the bends in the flow passage will have an insignificant effect compared with that of the mean flow. Hence the forces exerted by

the fluid are of two kinds First, there are transverse forces which arise from displacements, this is in effect due to a linear momentum flux tangential to the pipe. These forces include the coriolis reactions and reactions due to transverse acceleration of the fluid mass Secondly, there are forces due to fluid friction This category must include both the tangential shear forces and normal forces due to the pressure developed in the fluid as a result of friction The important conclusion is that these forces have a neutral total effect on the pipe. Thus the dynamical problem is independent of fluid friction

An exhaustive literature survey was given by Gregory and [1] Paidoussis [14] and Paidoussis and Issid [15] from which we see a remarkable development of the subject in the twenty years since the Trans - Arabian pipeline was observed to vibrate presumably as a result of internal flow However, the impetus for most of these studies did not come from a desire to solve a practical problem, but rather from one or more of the following considerations

- (i) the physical problem was inherently intriguing,
  - (ii) the problem offered scope for interesting mathematical manipulation,
  - (iii) the problem represents one of the few cases where a (non-conservative) follower load is physically realizable.
- The force of this last point becomes obvious upon recalling the very considerable theoretical interest

in the stability of column subjected to a tangential follower load at the free end [ 16 ], even when no means was evident for producing it Now, internal flow existing from the free end of a cantilever does generate a tangential load This being a physically easily realizable system - as compared to using solid - fuel rockets Quite recently, however, increasing industrial interest has developed in the dynamics of pipes conveying fluid, in conjunction with determining the response of such systems to arbitrary force fields, with application to heat exchanger, liquid-fuel rocket piping and nuclear reactor coolant channels

### 1.5 Effect of Transient Follower Load

S.T. Nouh and V. Sundararajan [ 17 ] have studied the amplification of the lateral response of a cantilever column assuming a set of initial perturbations, effect of pulse duration, internal damping and the effect of different initial perturbations, have also been discussed.

### 1.6 Present Work

The response of a linear elastic cantilevered system subjected to a nonconservative load has been dealt with in this thesis The following specific problems have been studied

1. Cantilever column subjected to a tangential follower load The effect of external and internal damping

on the first critical follower load have been studied. The combined effect of both dampings on critical load is also discussed.

- 2 Cantilever pipe conveying fluid under tangential follower load. The effect of fluid flow, coriolis force due to it and internal damping on external tangential follower load (less than the first critical load 20°05) have been discussed.
- 3 Cantilever column under transient follower load. The effect of very high loads (several times the first critical load), acting for a short duration, on the response of the column is discussed. The effect of load duration on the response of column is also studied.
4. Cantilever pipe conveying fluid under transient tangential follower load. The effect of fluid flow, coriolis force due to it and internal damping on the response of the pipe subjected to high loads (several times the first critical load), is studied.

The equation of motion is found by equilibrium method. In the present work the stability problem is discussed from the response of the cantilever column. The Galerkin's technique is used for reducing the linear partial differential equation to ordinary linear differential equation and Haming's method is used for performing the solution of coupled linear differential equations.

## CHAPTER 2

### FORMULATION

In this chapter the equation of motion of a cantilever pipe conveying fluid under external follower load is formulated by the equilibrium method

#### 2.1 Equation of motion

A uniform cantilever pipe of length  $L$ , flexural rigidity  $EI$  and mass per unit length  $m_p$ , conveying a stream of incompressible fluid of mass  $m_f$  per unit length with a mean flow velocity  $U$  and a follower force  $P_0$  acting at its free end is shown in Fig 2 (a) The  $x$ -axis coincides with the centre line of the undeflected pipe and its transverse deflection  $y$  is measured perpendicular to  $ox$ . Both  $ox$  and  $oy$  lie in a horizontal plane

The free body diagrams of fluid element and a pipe element are shown in Fig 2 (b - c)

The cross sectional area through which the fluid passes is  $A$ , the internal perimeter is  $S$  and the internal pressure is  $p$ . Consider now a small element of the pipe, of length  $\delta x$ , and the corresponding element of the enclosed fluid, of volume  $\delta D$ . The rate of change of momentum over  $\delta D$  may be written as

$$\frac{dM}{dt} = \iiint_{\delta D} [\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v}] \rho dD \quad (2.1)$$

where  $\vec{v}$  is the instantaneous velocity of a small element

$dD$  within  $\delta D$  Assuming that the radial variations in the flow velocity are small and that secondary flow effects are negligible - which was actually presupposed in the assumption of planar motions that is, assuming that one has plug flow and hence  $\vec{v}$  may be written as

$$\vec{v} = \vec{U} + \frac{\partial y}{\partial t} \hat{i} + \frac{\partial y}{\partial x} \hat{j} \quad (2.2)$$

Moreover, upon assuming that  $y$  and  $\frac{\partial y}{\partial t}$  are small and that motions are of long wavelength such that  $\frac{\partial y}{\partial x}$  is small, Equations (2.1) and (2.2) after some manipulation yield

$$\frac{dH}{dt} = m_f \frac{\partial U}{\partial t} \delta, \hat{i} + m_f \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \left( \frac{\partial y}{\partial t} + U \frac{\partial y}{\partial x} \right) \delta x \hat{j} \quad (2.3)$$

Hence, for the fluid element of Fig 2 (b), force balances in the  $x$  and  $y$  directions yield

$$A \frac{\partial p}{\partial x} + q_s S = 0 \quad (2.4)$$

$$F - A \frac{\partial}{\partial x} \left( p \frac{\partial y}{\partial x} \right) - m_f \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 y = 0 \quad (2.5)$$

where  $q_s$  is the shear stress on the internal surface of the pipe and  $F$  the transverse force per unit length between pipe wall and fluid.

The equations of motion for the pipe may similarly be found as

$$\frac{\partial T}{\partial x} + q_s S - Q \frac{\partial^2 y}{\partial x^2} = 0 \quad (2.6)$$

and

$$-P_0 \frac{\partial^2 y}{\partial x^2} - K \frac{\partial y}{\partial t} + \frac{\partial Q}{\partial x} + T \frac{\partial^2 y}{\partial x^2} - F - m_p \frac{\partial^2 y}{\partial t^2} = 0 \quad (2.7)$$

where  $P_0$  is the external follower load acting at the free end of the cantilever,  $T$  is the longitudinal tension,  $Q$  is the transverse shear force in the tube and  $K$  is the coefficient of viscous damping due to friction of the pipe with the surrounding stationary fluid medium. The shear force  $Q$  is related to bending moment  $M_b$  acting on the section of the tube by

$$\frac{\partial M_b}{\partial x} + Q = 0 \quad (2.8)$$

provided rotary inertia is neglected, while  $M_b$  is related to the lateral deflection by

$$M_b = E I \frac{\partial^2 y}{\partial x^2} \quad (2.9)$$

From Equations (2.8) and (2.9), we have

$$Q = -\frac{\partial M_b}{\partial x} = -E I \frac{\partial^3 y}{\partial x^3} \quad (2.10)$$

We consider  $E^*$  as the coefficient of internal dissipation which was assumed to be viscoelastic and of the Kelvin - Voigt type. Then

$$Q = -\left(E^* \frac{\partial}{\partial t} + E\right) I \frac{\partial^3 y}{\partial x^3} \quad (2.11)$$

The terms of second order magnitude are neglected in accordance with the Euler - Bernoulli beam approximation for small lateral motions. On combining Equations (2.5), (2.7) and (2.11), we have

$$\begin{aligned}
 & \left( E^* \frac{\partial}{\partial t} + E \right) I \frac{\partial^4 y}{\partial x^4} - \frac{\partial}{\partial x} \left\{ (-T + p A) \frac{\partial^3 y}{\partial x^3} \right\} \\
 & + m_f \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 y + K \frac{\partial^2 y}{\partial t^2} + m_p \frac{\partial^2 y}{\partial t^2} + P_o \frac{\partial^2 y}{\partial x^2} = 0
 \end{aligned} \tag{2.12}$$

$q_s$  may be eliminated between Eq (2.4) and Eq (2.6) to give

$$\frac{\partial}{\partial x} (p A - T) = 0 \tag{2.13}$$

Thus  $(p A - T)$  is independent of  $x$ , furthermore, at the free end  $p A - T = 0$  and consequently

$$p A - T = 0 \tag{2.14}$$

everywhere. This is an important conclusion since it establishes that the dynamical problem is independent of fluid friction.

On substituting Eq (2.14) into Eq. (2.12), we obtain

$$\begin{aligned}
 & \left( E^* \frac{\partial}{\partial t} + E \right) I \frac{\partial^4 y}{\partial x^4} + m_f \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 y \\
 & + K \frac{\partial^2 y}{\partial t^2} + m_p \frac{\partial^2 y}{\partial t^2} + P_o \frac{\partial^2 y}{\partial x^2} = 0
 \end{aligned} \tag{2.15}$$

i.e.,

$$\begin{aligned}
 & I E^* \frac{\partial^5 y}{\partial x^4 \partial t} + E I \frac{\partial^4 y}{\partial x^4} + (P_o + U^2) \frac{\partial^2 y}{\partial x^2} \\
 & + 2 m_f U \frac{\partial^2 y}{\partial x \partial t} + K \frac{\partial^2 y}{\partial t^2} + (m_p + m_f) \frac{\partial^2 y}{\partial t^2} = 0
 \end{aligned} \tag{2.16}$$

This is the required equation for studying the response of a cantilever pipe conveying fluid under follower force with the boundary conditions as follows

$$y = \frac{\partial y}{\partial x} = 0 \quad \text{at} \quad x = 0$$

and (2.17)

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^3 y}{\partial x^3} = 0 \quad \text{at} \quad x = L$$

## 2.2 Nondimensionalization of the Equation of Motion

Before we proceed with the analysis, it is desirable to express the problem in dimensionless parameters and accordingly we choose

$$\xi = \frac{x}{L}, \quad w = \frac{y}{L}, \quad \tau = \left( \frac{EI}{m_f + m_p} \right)^{1/2} \frac{t}{L^2},$$

$$u = \left( \frac{m_f}{EI} \right)^{1/2} UL, \quad r^2 = \frac{m_f}{m_f + m_p},$$

$$\gamma = \frac{E^*}{L^2} \left( \frac{I}{EI(m_f + m_p)} \right)^{1/2}, \quad c = \frac{KL^2}{2(EI(m_f + m_p))^{1/2}},$$

and

$$P(\tau) = \frac{P_0 L^2}{EI} \geq 0 \quad \text{in } (0, \tau) \quad \text{and} \quad P(\tau) = 0 \quad \text{when} \\ \tau > \tau^*, \quad \text{where } \tau^* \text{ is the load duration}$$

Now Equation (2.16) can be written in terms of dimensionless parameters as

$$\begin{aligned} & \frac{\partial^4 w}{\partial \xi^4} + \gamma \frac{\partial^5 w}{\partial \xi^4 \partial \tau} + \{ P(\tau) + u^2 \} \frac{\partial^2 w}{\partial \xi^2} \\ & + 2ur \frac{\partial^2 w}{\partial \xi \partial \tau} + 2c \frac{\partial w}{\partial \tau} + \frac{\partial^2 w}{\partial \tau^2} = 0 \end{aligned} \quad (2.18)$$

and boundary conditions reduce to

$$\begin{aligned} w &= \frac{\partial w}{\partial \xi} = 0 && \text{at } \xi = 0 \\ \frac{\partial^2 w}{\partial \xi^2} &= \frac{\partial^3 w}{\partial \xi^3} = 0 && \text{at } \xi = 1 \end{aligned} \tag{2.19}$$

## CHAPTER 3

### SOLUTION

The solution procedure for Eq (2.18) is discussed in this chapter. A brief computational approach for solving a set of ordinary coupled differential equations is also given.

#### 3.1 Solution Procedure

Equations (2.18) are reduced to an infinite system of ordinary differential equations through a Galerkin's procedure. We now apply the Galerkin's method for the analysis of system (2.18). We consider a set of orthonormal eigenfunctions  $\{\phi_m(\xi)\}$ , obtained by solving the following eigenvalue problem

$$\begin{aligned} \frac{d^4 \phi_m}{\xi^4} - \omega_m^2 \phi_m &= 0 \\ \phi_m - \frac{d \phi_m}{d \xi} &= 0 \quad \text{at } \xi = 0 \\ \frac{d^2 \phi_m}{d \xi^2} = \frac{d^3 \phi_m}{d \xi^3} &= 0 \quad \text{at } \xi = 1 \end{aligned} \tag{3.1}$$

The orthonormal eigenfunction or characteristic function obtained from equation (3.1) is given below

$$\begin{aligned} \phi_m(\xi) &= \{ \cosh(\beta_m \xi) - \cos(\beta_m \xi) - \alpha_m \\ &\quad [ \sinh(\beta_m \xi) - \sin(\beta_m \xi) ] \} \end{aligned} \tag{3.2}$$

where  $\alpha_m = \frac{\cos(\beta_m) + \cosh(\beta_m)}{\sin(\beta_m) + \sinh(\beta_m)}$ ,  $\omega_m^2 = \beta_m^2$

and  $\beta_m$  satisfies the frequency or characteristic equation

$$\text{Cosh}(\beta_m) \text{Cos}(\beta_m) = -1$$

The values of  $\beta_m$  and  $\alpha_m$  are given in a tabular form in Appendix (A) for various values of  $m$

Let us assume a solution of the form

$$w(\xi, \tau) = \sum_{n=1}^{\infty} q_n(\tau) \phi_n(\xi) \quad (3.3)$$

and variation in  $w$  of the form

$$\delta w = \sum_{m=1}^{\infty} \phi_m(\xi) \delta q_m(\tau) \quad (3.4)$$

for Eq (2.18).

Substituting Eq. (3.3) into Eq (2.18) and multiplying both sides by  $\delta w$ , we get

$$\begin{aligned} & \sum_{n=1}^{\infty} \frac{d^4 \phi_n}{d \xi^4} q_n(\tau) \delta w + \{ P(\tau) + u^2 \} \sum_{n=1}^{\infty} \frac{d^2 \phi_n}{d \xi^2} q_n(\tau) \delta w \\ & + \sum_{n=1}^{\infty} \phi_n(\xi) \dot{q}_n(\tau) \delta w + \sum_{n=1}^{\infty} \gamma \frac{d^4 \phi_n}{d \xi^4} q_n(\tau) \delta w \\ & + 2 u r \sum_{n=1}^{\infty} \frac{d \phi_n}{d \xi} q_n(\tau) \delta w + 2 c \sum_{n=1}^{\infty} \phi_n(\xi) \dot{q}_n(\tau) \delta w \\ & = 0 \quad (3.5) \end{aligned}$$

where a dot indicates the differentiation with respect to  $\tau$

Substituting Eq. (3.4) into Eq (3.5) and integrating from zero to unity we get

$$\begin{aligned}
& \int_0^1 \sum_{n=1}^{\infty} \frac{d^4 \phi_n}{d \xi^4} q_n(\tau) \sum_{m=1}^{\infty} \phi_m(\xi) \delta q_m(\tau) d\xi \\
& + \int_0^1 \{ P(\tau) + u^2 \} \sum_{n=1}^{\infty} \frac{d^2 \phi_n}{d \xi^2} q_n(\tau) \sum_{m=1}^{\infty} \phi_m(\xi) \delta q_m(\tau) d\xi \\
& + \int_0^1 \sum_{n=1}^{\infty} \phi_n(\xi) q_n(\tau) \sum_{m=1}^{\infty} \phi_m(\xi) \delta q_m(\tau) d\xi \\
& + \int_0^1 \gamma \sum_{n=1}^{\infty} \frac{d^4 \phi_n}{d \xi^4} q_n(\tau) \sum_{m=1}^{\infty} \phi_m(\xi) \delta q_m(\tau) d\xi \\
& + \int_0^1 2 u r \sum_{n=1}^{\infty} \frac{d \phi_n}{d \xi} q_n(\tau) \sum_{m=1}^{\infty} \phi_m(\xi) \delta q_m(\tau) d\xi \\
& + 2 c \int_0^1 \sum_{n=1}^{\infty} \phi_n(\xi) q_n(\tau) \sum_{m=1}^{\infty} \phi_m(\xi) \delta q_m(\tau) d\xi = 0
\end{aligned}$$

or

$$\begin{aligned}
& q_m + \sum_{n=1}^{\infty} (\beta_m^4 \delta_{mn} + P(\tau) \phi_{mn} + u^2 \phi_{mn}) q_n \\
& + \sum_{n=1}^{\infty} 2 u r \psi_{mn} \dot{q}_n + \sum_{n=1}^{\infty} \gamma \beta_m^4 \delta_{mn} \dot{q}_n \\
& + 2 c \sum_{n=1}^{\infty} \delta_{mn} q_n = 0 \tag{36}
\end{aligned}$$

where

$$\begin{aligned}
\phi_{mn} &= \int_0^1 \phi_m(\xi) \frac{d^2 \phi_n}{d \xi^2} d\xi \quad \phi_{mn} \neq \phi_{nm} \text{ for } m \neq n \\
\psi_{mn} &= \int_0^1 \phi_m(\xi) \frac{d \phi_n}{d \xi} d\xi \quad \psi_{mn} \neq \psi_{nm} \text{ for } m \neq n \\
\delta_{mn} &= \int_0^1 \phi_m(\xi) \phi_n(\xi) d\xi = \begin{cases} 1 & \text{for } m = n \\ 0 & \text{for } m \neq n \end{cases}
\end{aligned}$$

Ultimately equation (3 6) reduces to

$$\begin{aligned} q_m + \{ P(\tau) + u^2 \} \sum_n \phi_{mn} q_n + \beta_m^4 q_m + \gamma \beta_m^4 q_m \\ + 2 u r \sum_n \psi_{mn} q_n + 2 c q_m = 0, \quad m = 1, 2, 3, \end{aligned} \quad (3 7)$$

where  $\beta_m$  are the roots of the frequency equation of the free undamped vibration of the beam

### 3.2 Computational Approach

Haming's method (Predictor - corrector method) [ 18 ] has been used for performing the solution of coupled Eqs. (3.7). Since this method solves a set of first order differential equations only, Eqs (3 7) are reduced to a set of first order differential equations. The reduction procedure is given in Appendix (B). Equation (3 7) contains two integrals  $\phi_{mn}$  and  $\psi_{mn}$  which are computed numerically as well as empirically. The empirical formulae for finding the integrals  $\phi_{mn}$  and  $\psi_{mn}$ , are given in Appendix (C).

The convergence of Galerkin's method for infinite system of Eqs. (3 7) is checked by increasing  $m$  and observing the asymptotic nature of the solution. The convergence of Haming's method is simultaneously checked by varying the integration step size and observing the asymptotic nature of the solution. The accuracy of the solution is seen up to fifth place of decimals. The double precision has been used for performing the solution of coupled equations.

## CHAPTER 4

### RESULTS AND DISCUSSION

The numerical solution of the coupled equations (3.7) is performed for the following cases

- 1 Effect of damping on critical follower load for a cantilever column
- 2 Cantilever pipe conveying fluid with follower load
- 3 Cantilever column under transient follower load
- 4 Cantilever pipe conveying fluid under transient follower load

The results obtained in the form of dimensionless parameters are discussed in this chapter. The effect of various dimensionless parameters on the response and magnification factor in the above cases have been discussed.

Initial conditions of the following form are assumed

$$q_1(0) = 10^{-4}, \quad q_m(0) = 0, \quad m > 1 \quad (4.1)$$

As a measure of the total response, the magnification factor  $M$  is introduced, defined by

$$M = \left\{ \sum q_m^2(\tau) \right\}^{1/2} / \left\{ \sum q_m^2(0) \right\}^{1/2} \quad (4.2)$$

The convergence of the Galerkin's procedure for a continuous system is checked by considering different values of  $m$ . The value of  $m = 5$  is finally used in the solution of Eqs (3.7). The coupled Eqs (3.7) are integrated numerically by Haming's method (Predictor -

Corrector Method) The convergence of Haming's method is checked by observing the numerical values up to the fifth place of decimals for two successive integration step sizes. The integration step size of 0.002 is finally used for performing the integration of Eqs (3.7) in all the above cases.

#### 4.1 Effect of Damping on Critical Follower Load ( $u = 0, r = 0$ )

It is well known that in the case of a cantilever column subjected to a tangential follower load, the critical value of the external load is 20.05. The corresponding critical load in the case of a conservative load is 25. The effect of external damping on the critical follower load is studied from the response behaviour of the column. The critical load increases with increase in the external load, which is 38 (Fig. 3). This result checks with that of Plaut and Infante [12] who have established this considering the problem as an eigenvalue problem. Thus the external damping stabilizes the system.

The effect of various values of internal damping on the critical load is shown in the Fig. 4. As internal damping increases the critical load decreases, and for a particular value of  $\gamma$  the critical load reduces by almost 60%. Thus the effect of internal damping is destabilizing, which is in contrast to that of external

damping For a given follower load, therefore, an increase in internal damping may cause a stable column to become unstable

Assuming the same value of internal and external dampings as 0.001 an attempt is made to explain the contrary behaviour of these two dampings The magnification factor with internal damping, external damping and without damping are given in Table 4.1 By closely studying the magnification factor it can be noticed that the effect of internal damping becomes more dominant after an elapse of time  $\tau$  as compared to that of external damping It seems as if the coupling due to  $\beta_m$  associated with the internal damping is responsible for the unusual behaviour of internal damping on critical follower load, whereas the external damping is not associated with any coupling of the modes

The combined effect of both external and internal dampings on critical follower load is shown in Table 4.2 With an internal damping of 0.01 the critical load decreases from 20.05 to 11.00 in absence of external damping. With a constant value of  $\gamma = 0.01$ , the increase in external damping ( $c = 1$ ) increases the critical load from 11.00 to 19.00. The reason for this behaviour of damping lies in the relationship between the restoring forces caused by internal and external damping, both of which depend on the rate motion but in different manners An increase in

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TABLE 4.1

The variation of magnification factor with time (at slightly less than the first critical load, 20) for internal damping, external damping and without damping is given below

$$u = r = 0$$

Time, $\tau$	M with $\gamma = 0.001$	M with $c = 0.001$	M with $\gamma = 0, c = 0$
0.4	2.046	2.079	2.079
0.8	1.696	1.419	1.420
1.2	2.772	3.892	3.897
1.6	6.503	6.029	6.038
2.0	4.098	0.996	0.999
2.4	3.844	8.306	8.326
2.8	11.087	4.386	4.398
3.2	10.304	5.760	5.780
3.6	1.633	7.544	7.571
4.0	17.721	0.559	0.562
4.4	23.533	5.902	5.928
4.8	7.375	2.749	2.762
5.2	25.060	1.497	1.505
5.6	47.114	1.348	1.355
6.0	30.371	1.136	1.143
6.4	27.301	2.788	2.806
6.8	84.913	0.990	0.996
7.2	81.797	5.032	5.069
7.6	11.077	5.714	5.757
8.0	137.365	2.602	2.624

TABLE 4 2

The effect of external damping on critical follower load  
keeping the internal damping constant, is shown below

$$u = r = 0$$

Internal damping	External damping	Critical follower load
0.01	0.0	11.0
0.01	0.01	11.5
0.01	0.10	14.0
0.01	1.00	19.0

external damping may decrease the effect of internal damping and the total restoring force on the column may be reduced

#### 4.2 Cantilever Pipe Conveying Fluid with Follower Load

In this case the effect of fluid flow and internal damping on external tangential follower load (less than the first critical load 20.05) have been discussed. The load is assumed to be 15.00. The variation of the magnification factor with time is shown in Fig. 5 for the cases with and without fluid flow and internal damping. In the absence of fluid flow ( $u = 0$ ) the internal damping destabilizes the system, which should be the case since the load is greater than first critical load with  $\gamma = 0.01$ . For different values of  $u$  the effect of flowing fluid on  $M$  is shown in Fig. 5. With a fluid flow of  $u = 1$  it can be seen that the system is damped down and the magnification factor decreases rapidly with time. Thus the fluid damping and coriolis force due to it, overcome the destabilizing effect of internal damping and stabilize the system.

There exists a critical fluid velocity, for a particular value of the mass ratio and external load, for which the system becomes unstable. For example when the mass ratio is 0.295 and the external load is 15.00 the critical fluid velocity is 4.5

#### 4.3 Cantilever Column Under Transient Follower Load ( $u = 0$ , $\gamma = 0$ , $c = 0$ )

The effect of load duration on the response of a cantilever column under a transient follower load of magnitude  $P = 25$  is shown in Fig. 6, 7. Since the response is dominated by first mode, only the response  $q_1(\tau) / q_1(0)$  is plotted against time  $\tau$ . The duration for which the load acts is denoted by  $\tau^*$ . It can be seen that the maximum amplification of the initial disturbance depends on the time at which the load is released and always occurs after the release of the load. For a load duration from 0.9 to 0.975 the response decreases with increase in load duration. As we increase the load from 0.975 the response starts increasing (Fig. 7).

The variation of the maximum magnification factor ( $M^*$ ) with the load duration  $\tau^*$  is shown in Fig. 8. Whether or not the maximum response increases or decreases as the load duration increases seems to depend on the deflection shape of the curve at the instant the load is released. This can be seen from Figs. 9 (a - d).

The effect of the load duration on the magnification factor in the case of a constant external impulse of magnitude 21 is shown in Fig. 10. With the load duration  $\tau^* = 0.3$ ,  $M^*$  reaches a value of the order of  $10^3$ . With  $\tau^* = 0.5$ ,  $M^*$  decreases to 470 and with  $\tau^* = 1$  to 11. Thus, the maximum magnification factor decreases considerably as the load duration increases.

An interesting case has been plotted in Fig 11 which corresponds to  $P = 120$  and  $\tau^* = 0.2$ . In this case it can be seen that the second mode dominates the overall response even though the initial condition is assumed to contain the first mode only. This type of phenomenon may be expected in the case when the external load is very high (several times the first critical load) and the load duration is small as compared to the natural period of the column. The study of this behaviour is made by keeping the load duration constant (say  $\tau^* = 0.2$ ) and varying the external load. It can be seen from the Figs 11, 12, 13 and 14 that for values of  $P$  greater or equal to 120 ( $P \geq 120$ ) the second mode dominates the overall response of the column even though the initial condition is given in first mode only. However, for a particular load duration  $\tau^*$  there exists an upper limit of the load beyond which this phenomenon disappears. This can be seen from the Fig. 14 which corresponds to the load  $P = 135$ . For a given load duration it seems that there exists a range of load in which the second mode dominates the overall response of the column. At very high loads one should be aware of the responses in higher modes also even though the initial perturbation is assumed to contain the first mode only.

#### 4.4 Cantilever Pipe Conveying Fluid Under Transient Follower Load

We discuss in this case the response of a pipe conveying fluid under a transient tangential follower load greater than the first critical load acting for a short duration  $\tau$ . The effect of internal damping, flowing fluid, coriolis force due to flowing fluid and combined effect of all these have been shown in Fig. 15. The external load is assumed to be  $P = 70$  and the load acts for a duration of  $\tau' = 0.3$ . The maximum magnification factor ( $M^*$ ) when no internal damping is present ( $\gamma = 0$ ) is seen to be very high, which should be the case since the load is more than three times the first critical load  $M^*$  in this case is of the order of  $10^3$ . With  $\gamma = 0.1$ ,  $M^*$  decreases to a value of 430 indicating the effect of internal damping in reducing  $M^*$  in the case of a transient follower load. Whereas in the case of a steady load (slightly less than first critical load without damping) the internal damping destabilizes the system (that is, the magnification factor increases with time) and in case of a transient follower load many times the first critical load acting for a short duration the effect of internal damping is to reduce  $M^*$ . The effect of flowing fluid is to reduce  $M^*$  further. This can be attributed to effect of the centrifugal and coriolis forces due to the fluid flow. When both the internal damping as well as the flowing fluid are present, then  $M^*$  reduces considerably.

The effect of mass ratio  $r$  ( $\frac{\text{mass of fluid}}{\text{mass of fluid} + \text{mass of pipe}}$ ) on  $M^*$  is also shown in the Fig. 15. The increase in mass ratio will cause a decrease in  $M^*$ . It can be noticed that in the case of fluid damping,  $M^*$  occurs much quicker than that in the case of internal damping.

Figures 16 and 17 correspond to the case where the second mode dominates the overall response of the pipe even though the initial condition is assumed in the first mode only. The response  $q_n(\tau) / q_1(0)$  is plotted against  $\tau$  for different modes with and without flowing fluid. The load is assumed to be  $P = 120$  and load duration  $\tau^* = 0.175$ . The presence of flowing fluid ( $u = 1$ ) does not change the maximum response in the second mode. The second mode still dominates the overall response. All the other modes (including the first mode) die down rapidly compared with the decay of the second mode.

#### 4.5 Comment on the Magnification Factor

The magnification factor was defined to represent the overall response of the column and the maximum magnification factor gives an idea of the maximum response. But it should be pointed out that the maximum magnification factor does not always correspond to the maximum response. This can be seen from Fig 9(c). However in most of the cases the magnification factor is a good measure of the overall response.

## CHAPTER 5

### CONCLUSIONS

From the study of the response of a linear elastic cantilevered system subjected to an external follower load, the following conclusions are arrived at

1. The internal damping destabilizes the nonconservative system. The critical load decreases with increase in internal damping. The external damping stabilizes the nonconservative system. This behaviour is in marked contrast to that of the internally damped column and also to that of the conservative system (where external damping has no effect on the critical load). It seems as if the coupling due to  $\beta_m$  associated with the internal damping is responsible for the unusual behaviour of internal damping on critical follower load, whereas the external damping is not associated with any coupling of the modes.
2. In case of external follower load (slightly less than first critical load) the fluid damping and coriolis force due to it overcome the destabilizing effect of internal damping and stabilize the system. There exists a critical fluid velocity for a particular value of the mass ratio and external load, for which the system becomes unstable.

3. It is found that the maximum amplification of the initial disturbance depends on the time at which the load is released and always occurs after the release of the load. Whether or not the maximum response increases or decreases as the load duration increases seemed to depend on the deflection shape of the column at the instant the load is released. In case of high loads (several times the first critical load) the maximum magnification factor decreases considerably with increase in load duration.
4. An interesting case has been established, in which for a particular load duration a range of load exists for which the second mode dominates the overall response of the column, even though the initial perturbation is assumed to contain the first mode only. Thus it can be concluded that in case of very high loads (several times the first critical load) acting for a short duration one should be aware of the responses in higher modes also. In this case the effect of fluid flow and coriolis force due to fluid flow is also studied. The second mode still dominates the overall response. The response in other modes (including first mode) damped out faster as compared to the decay in second mode response.
5. The fluid damping, coriolis force due to it and material damping reduce the maximum magnification factor considerably as compared to the case with fluid or material

dampings alone In case of fluid damping  $M^*$  occurs quicker as compared to the case with the internal damping

- 6 The maximum magnification factor does not always correspond to the maximum response However, in most of the cases it is a good measure of the overall response

#### Future Work

The unusual behaviour of the internal damping on critical follower load should be studied rigorously Further thought should be given to this problem by considering the loss factor, strain energy etc of the system In case of very high loads (several times the first critical load) acting for a short duration, the second mode dominates the overall response for a particular range of loads A theoretical explanation is needed for this special behaviour The transient external follower load of other types such as triangular and blast type will be of interest

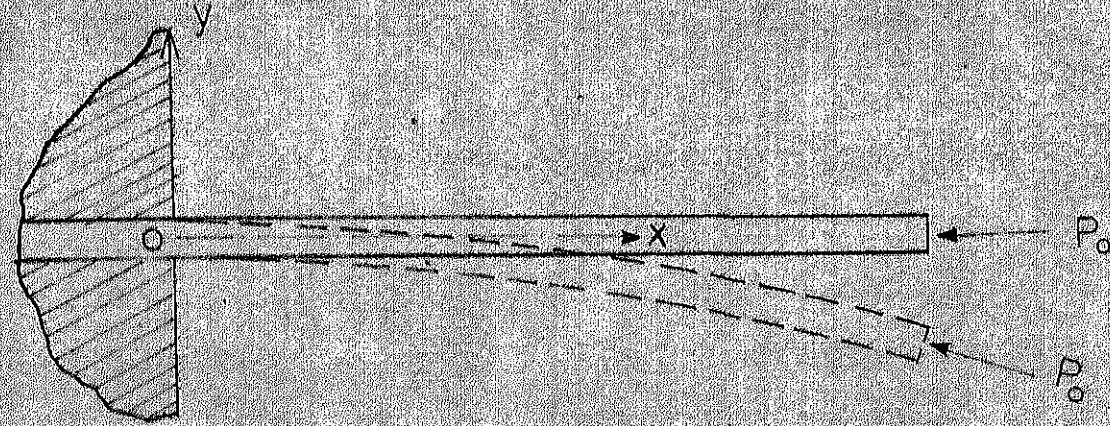


Fig. 2 (a) Cantilever pipe

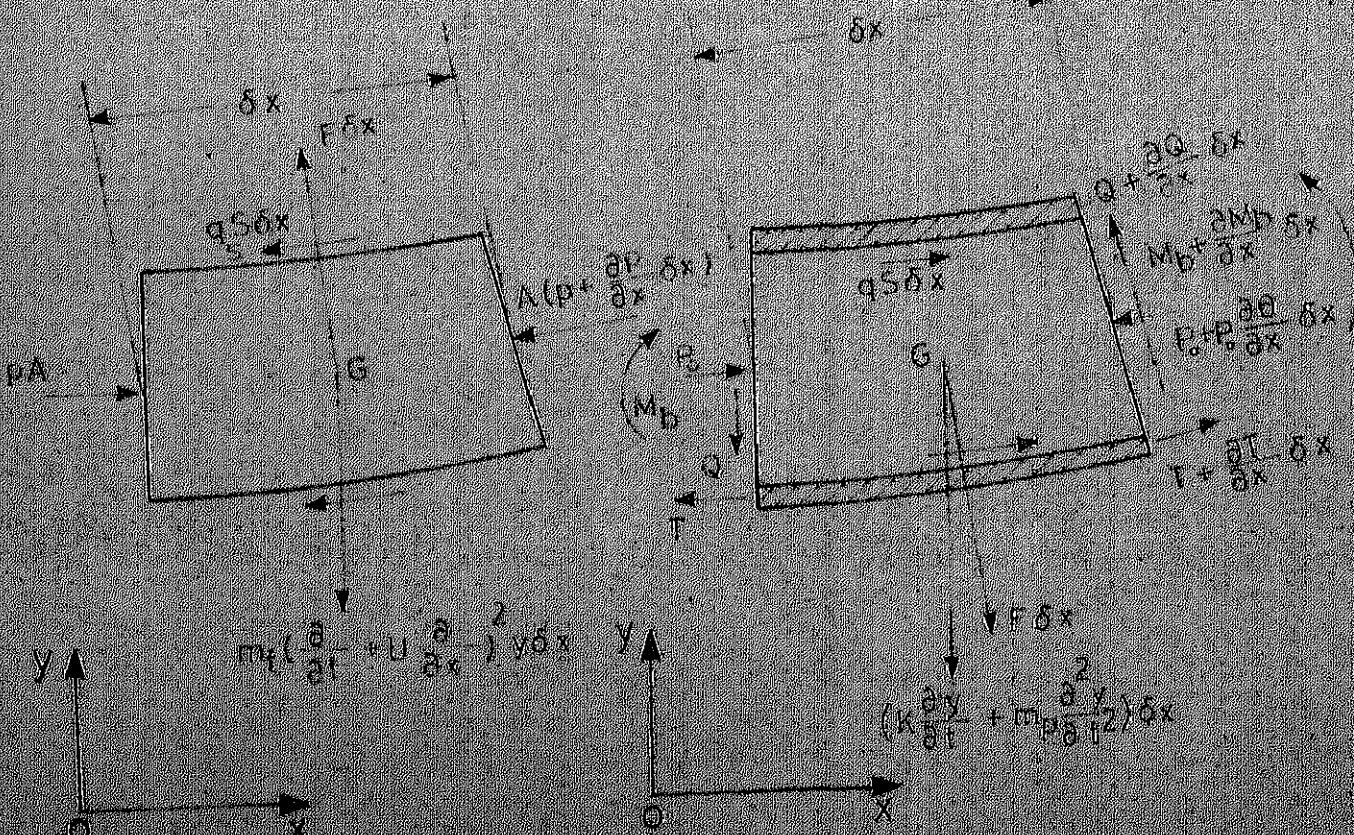


Fig. 2 (b) Fluid element    Fig. 2 (c) Pipe element

Fig. 2 Free body diagrams of pipe and fluid elements

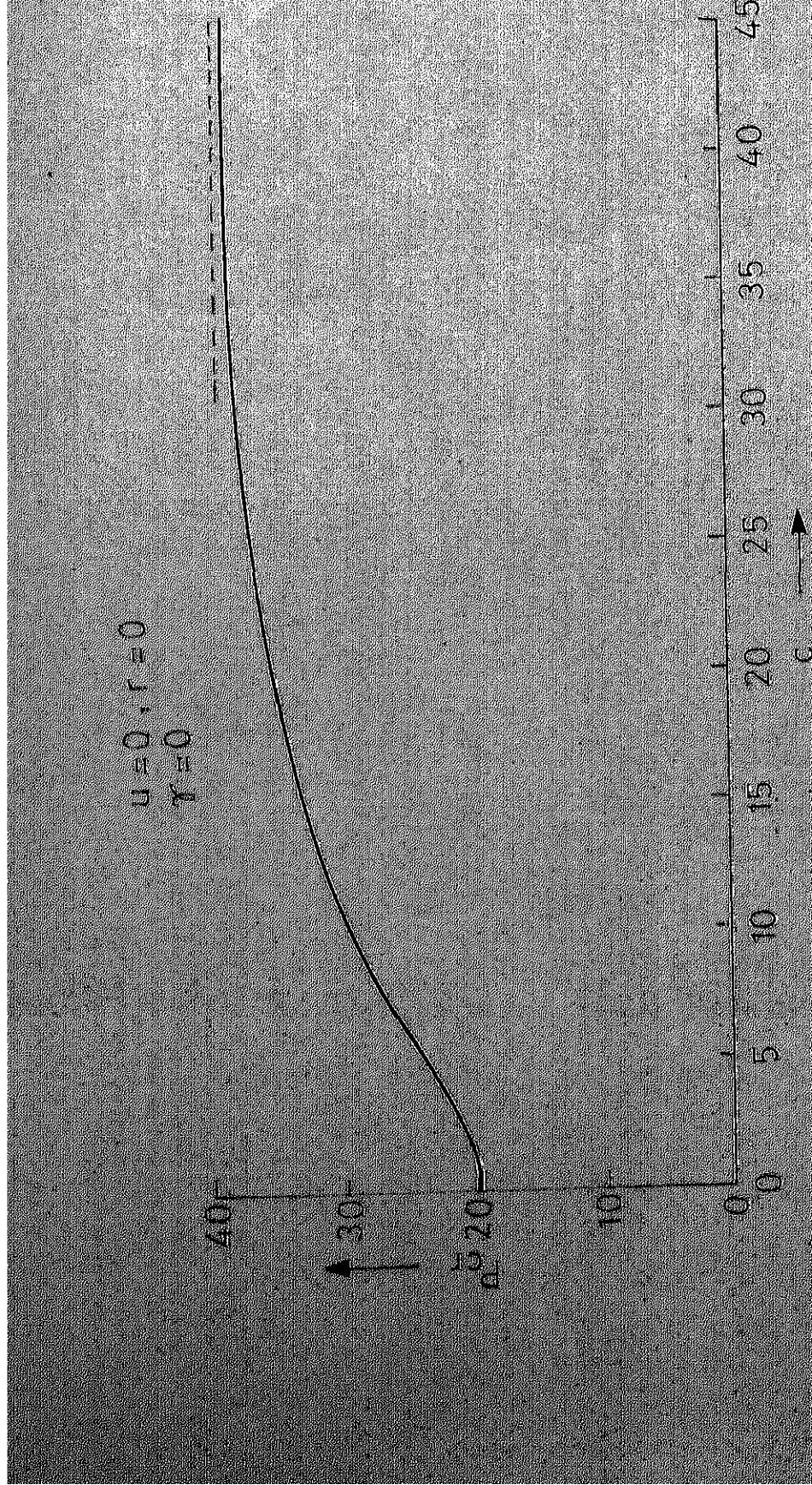


Fig. 3 Effect of external damping on external critical follower load

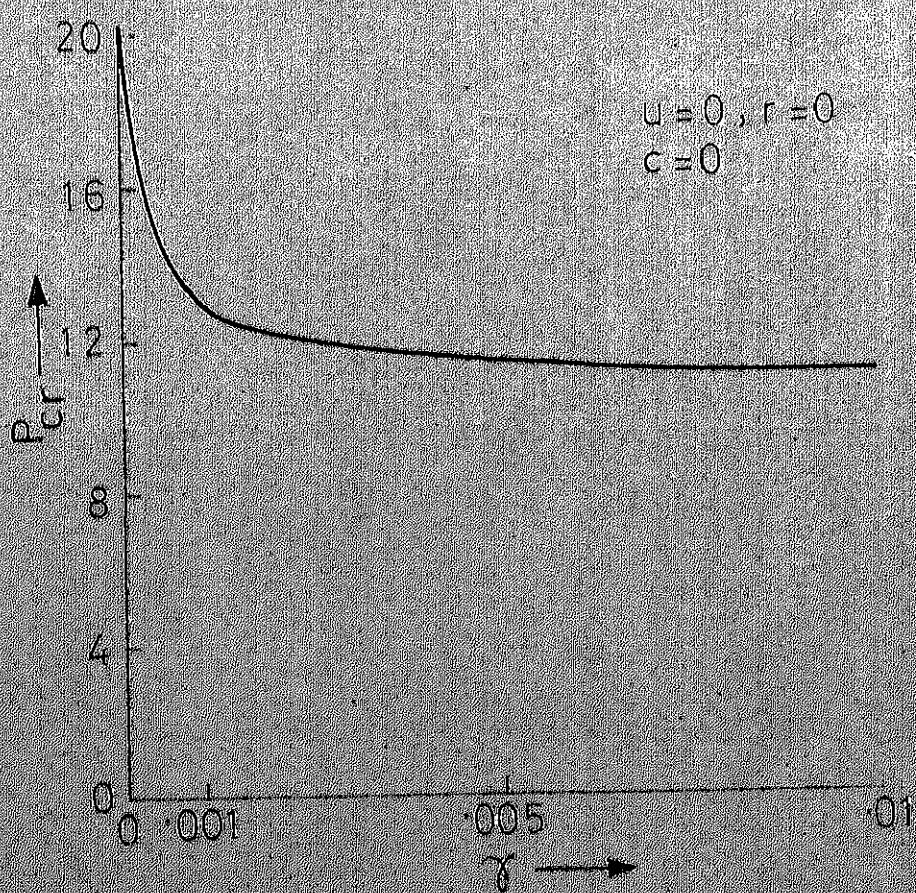


Fig.4 Effect of internal damping on first critical load

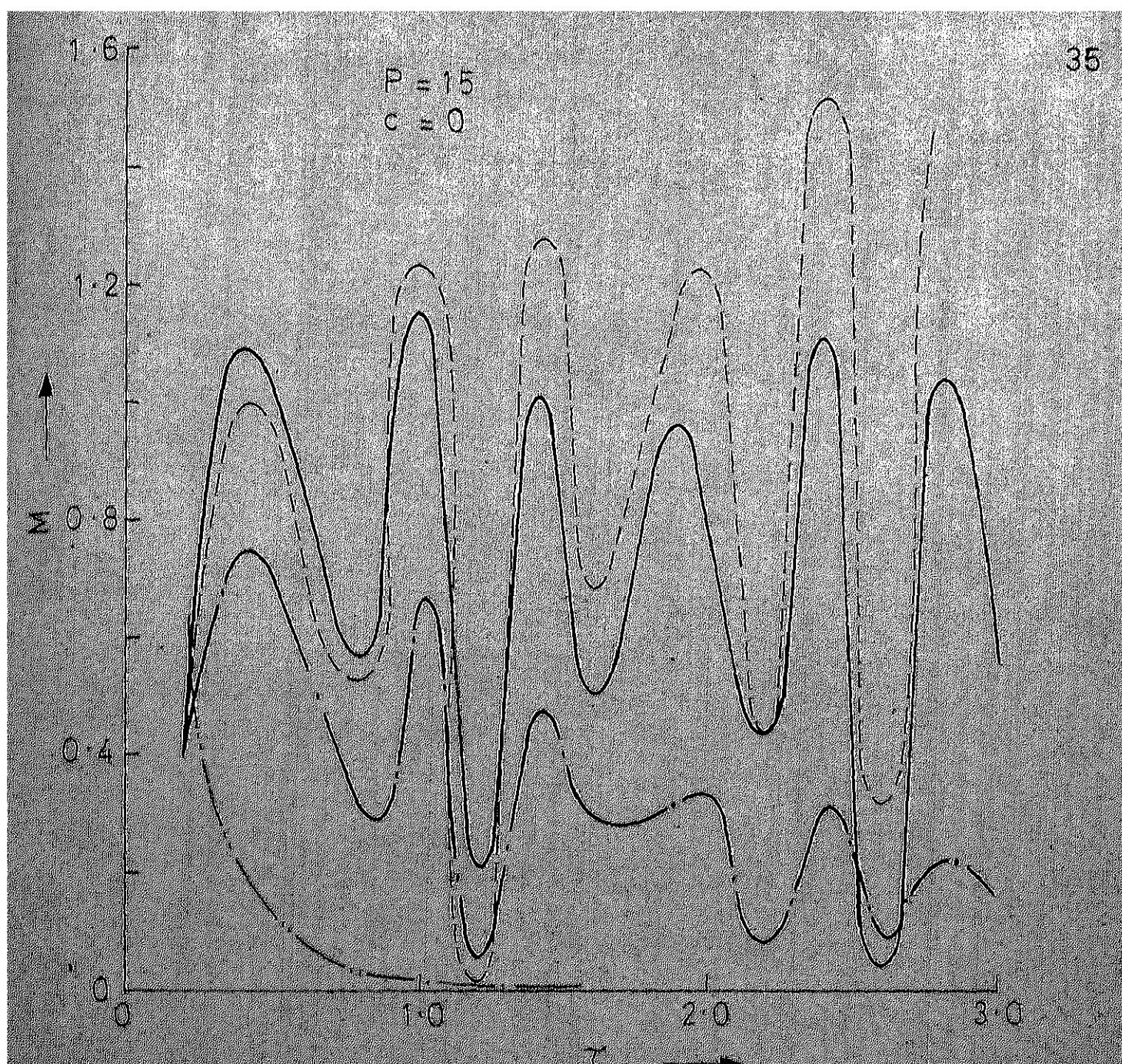


Fig. 5 Variation of magnification factor with time

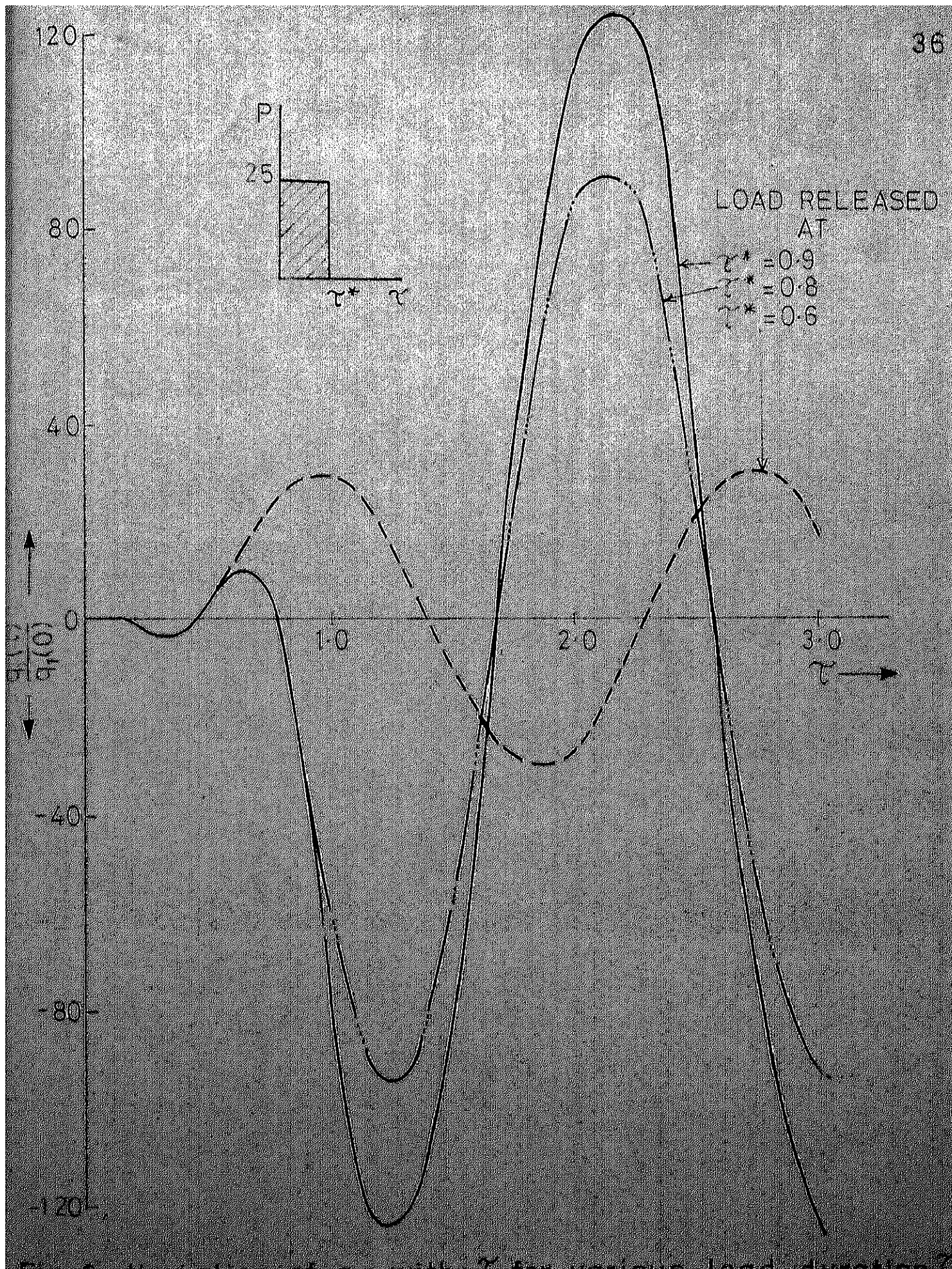


Fig. 6 Variation of  $q_1$  with  $T$  for various load duration

LOAD RELEASED  
AT

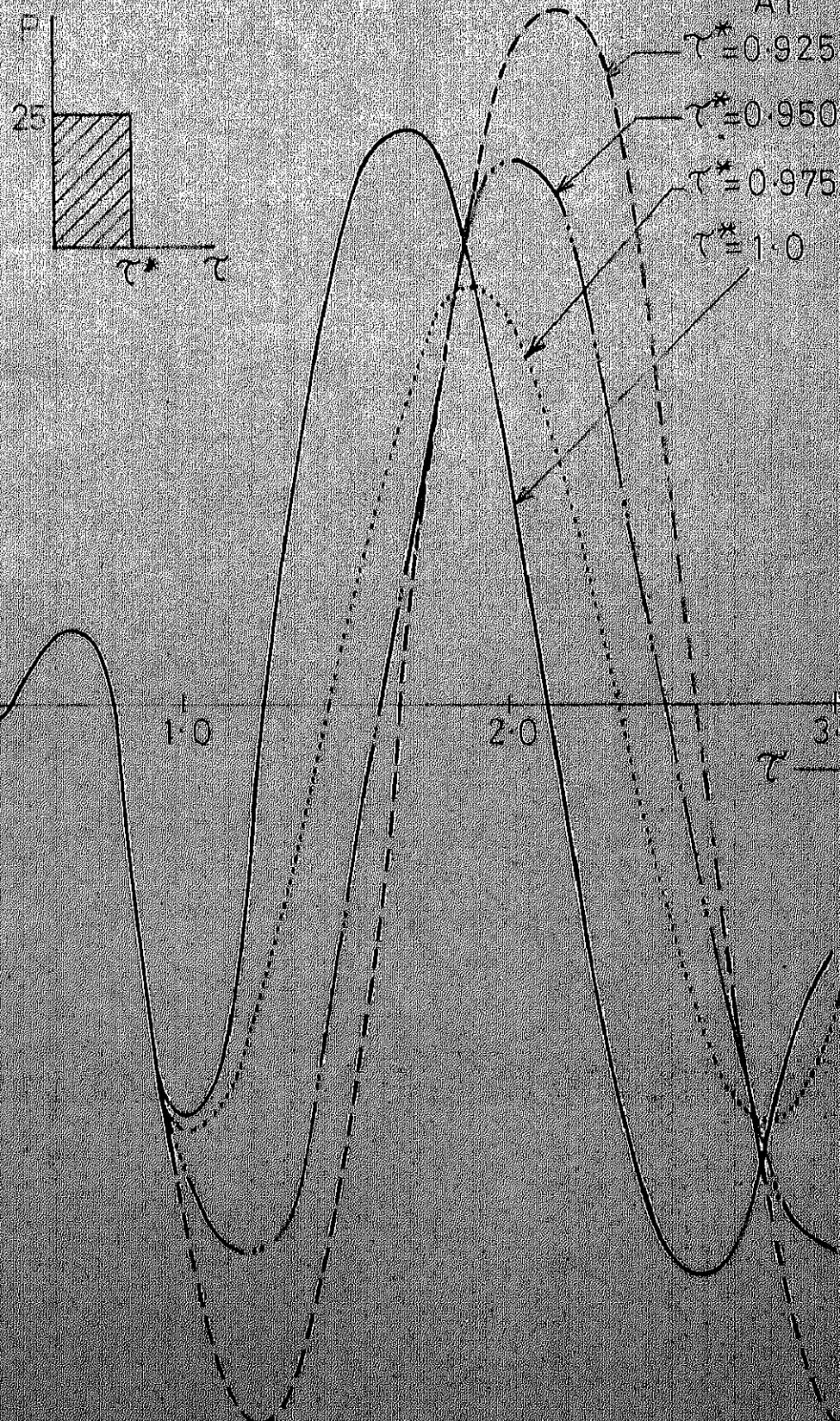


Fig. 7 Variation of  $q_1$  with  $\gamma$  for various load duration  $T$

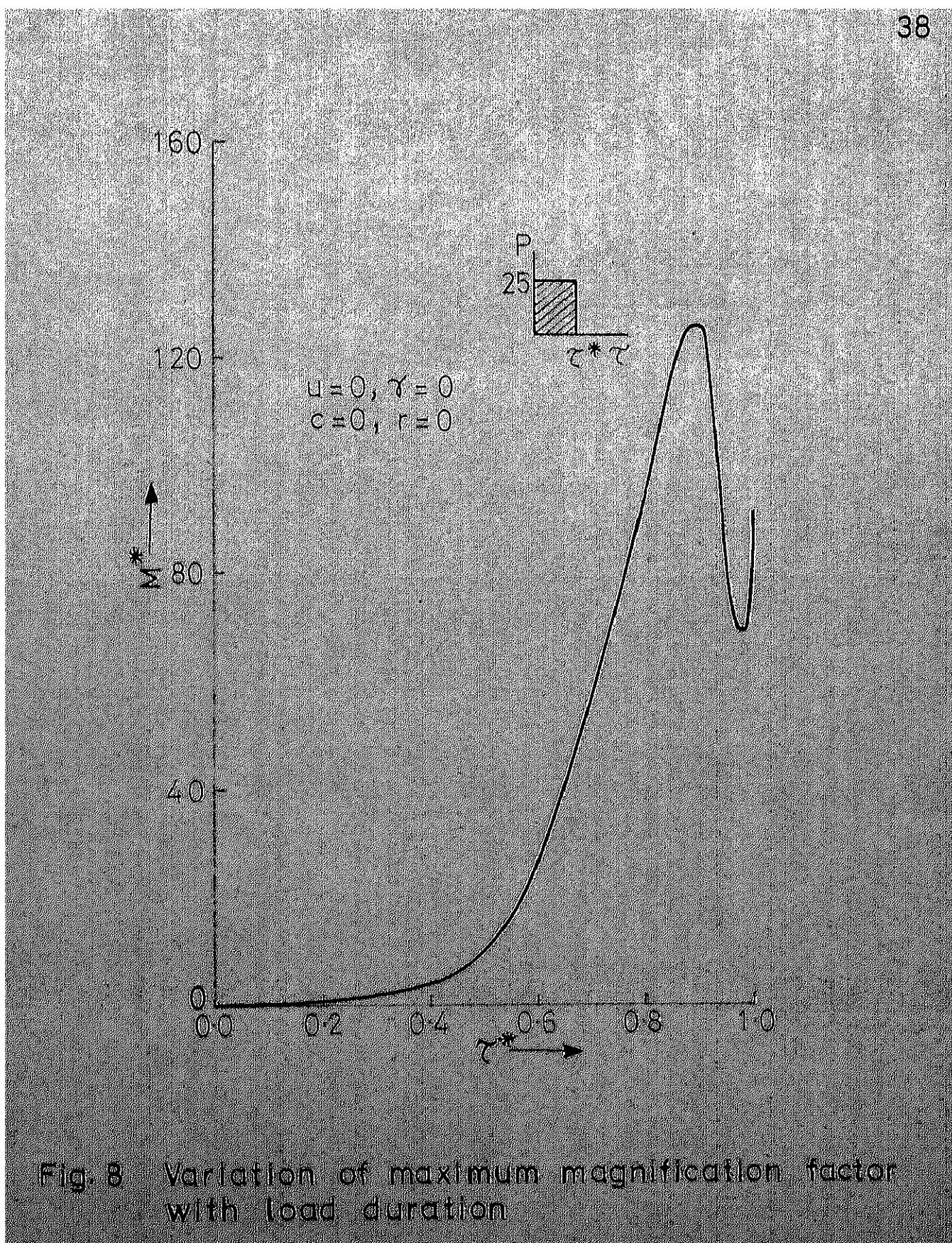


Fig. 8 Variation of maximum magnification factor with load duration

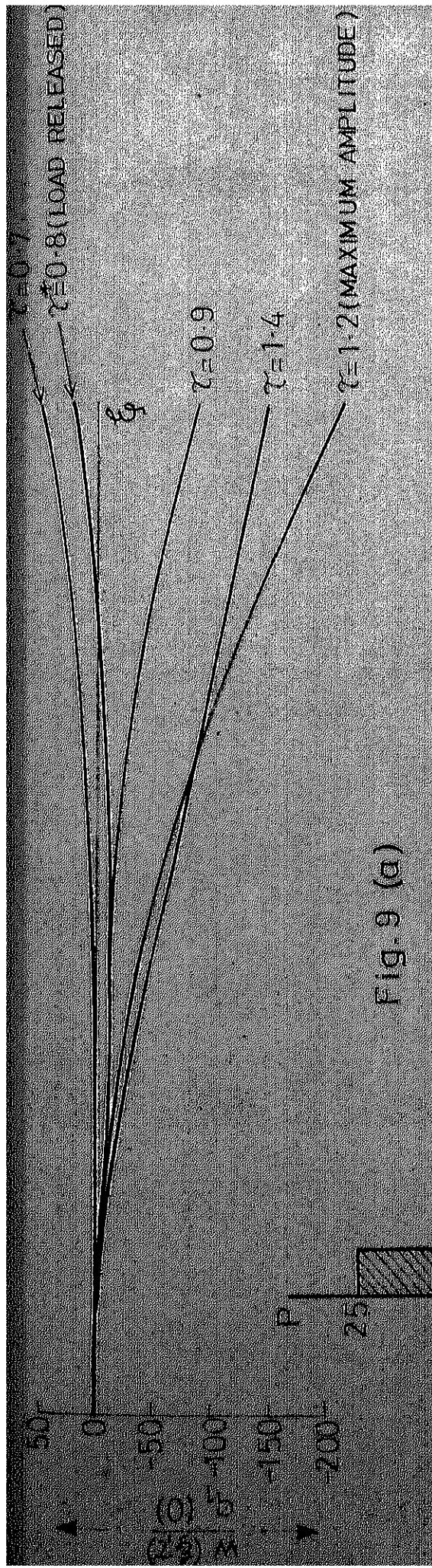


Fig. 9 (a)

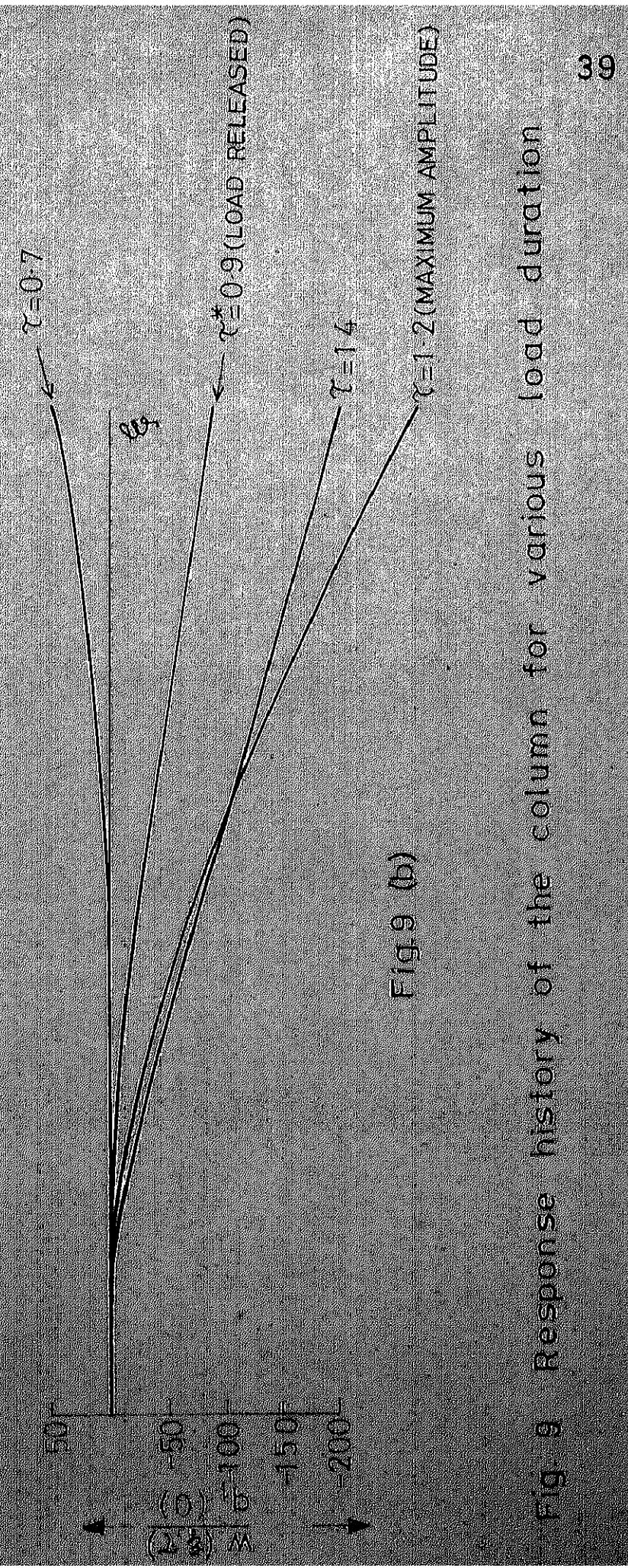


Fig. 9 (b)

Fig. 9 Response history of the column for various load duration

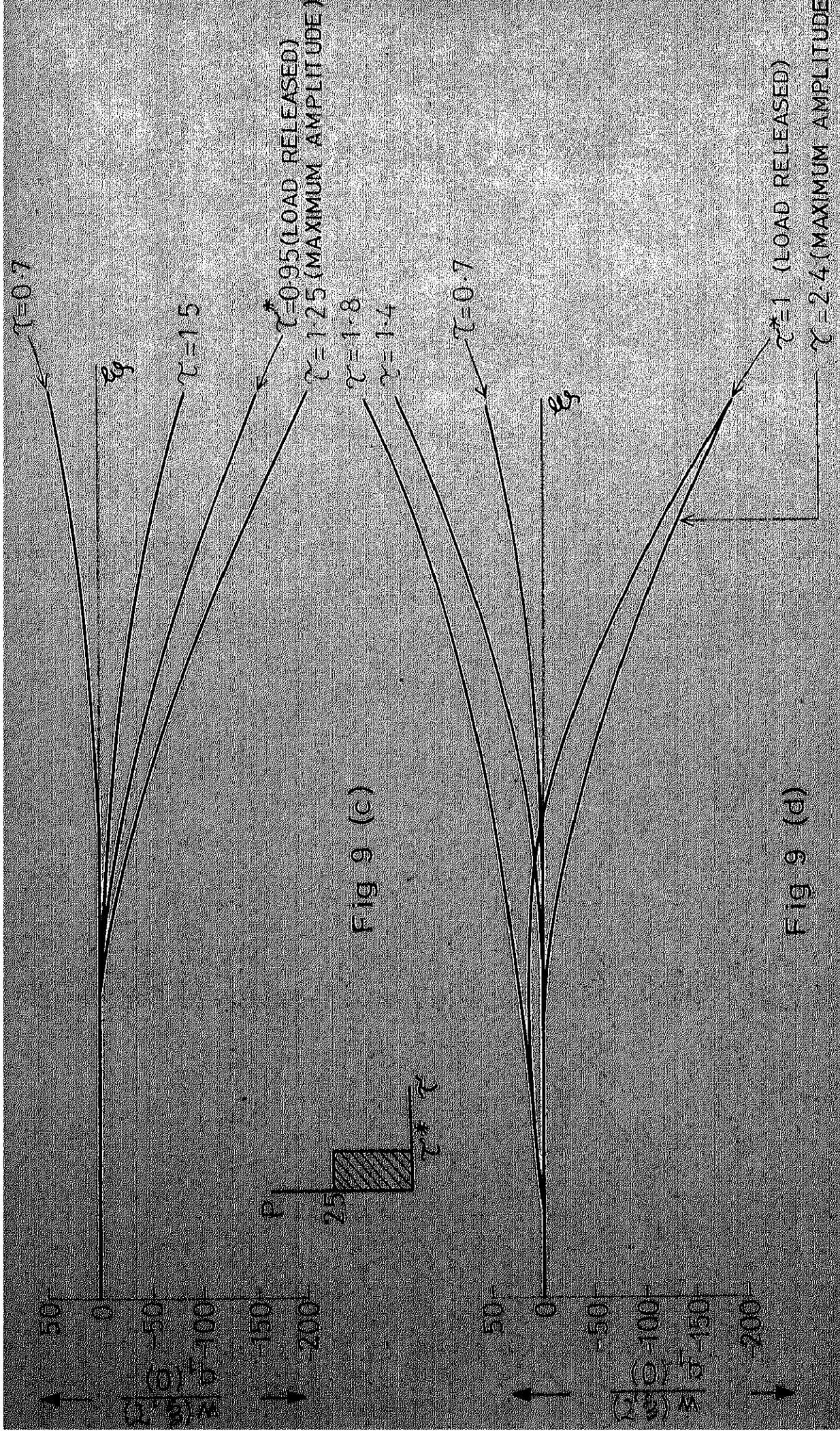


Fig. 9 Response history of the column for various load duration ratios

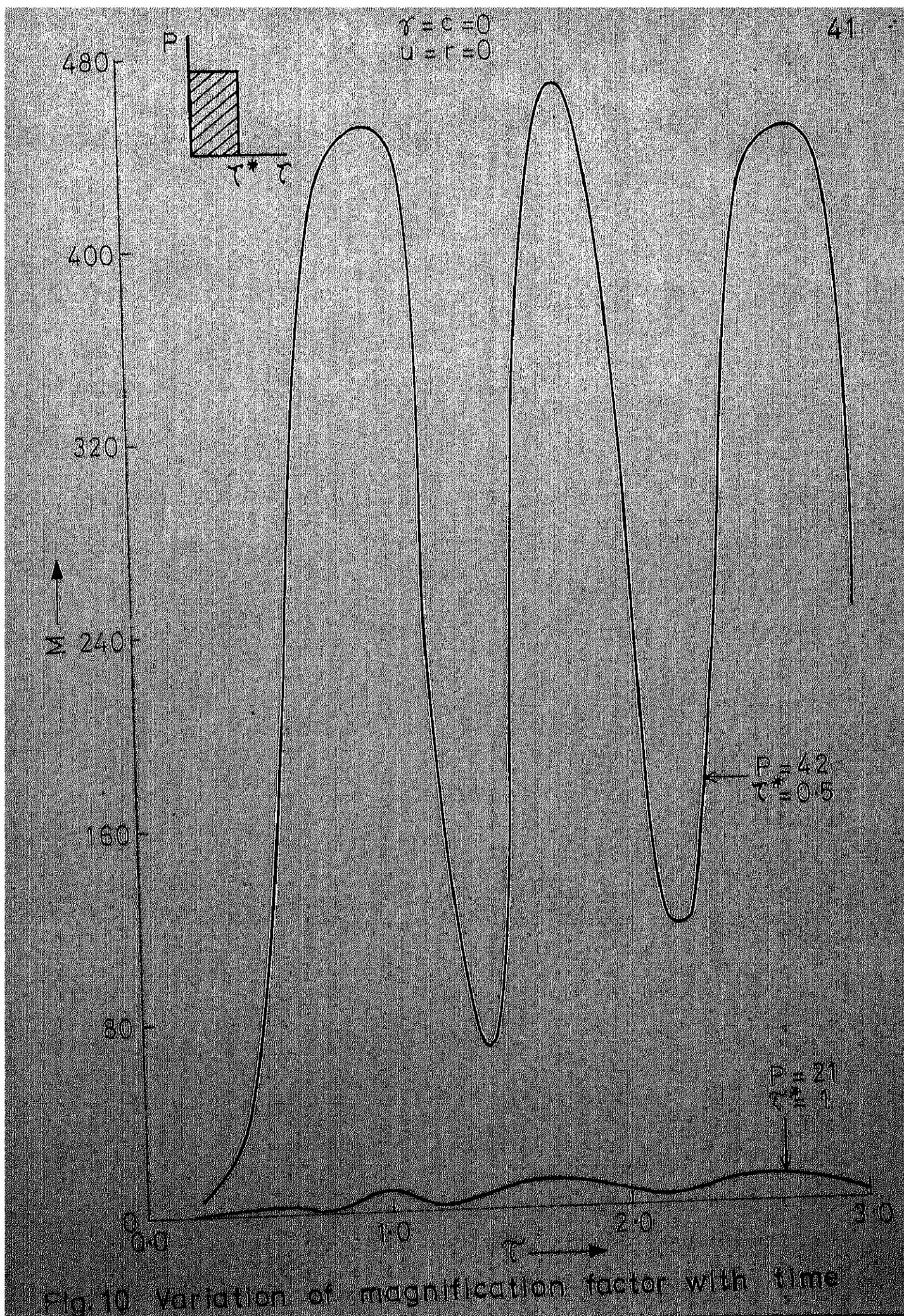


Fig. 10. Variation of magnetization factor with time

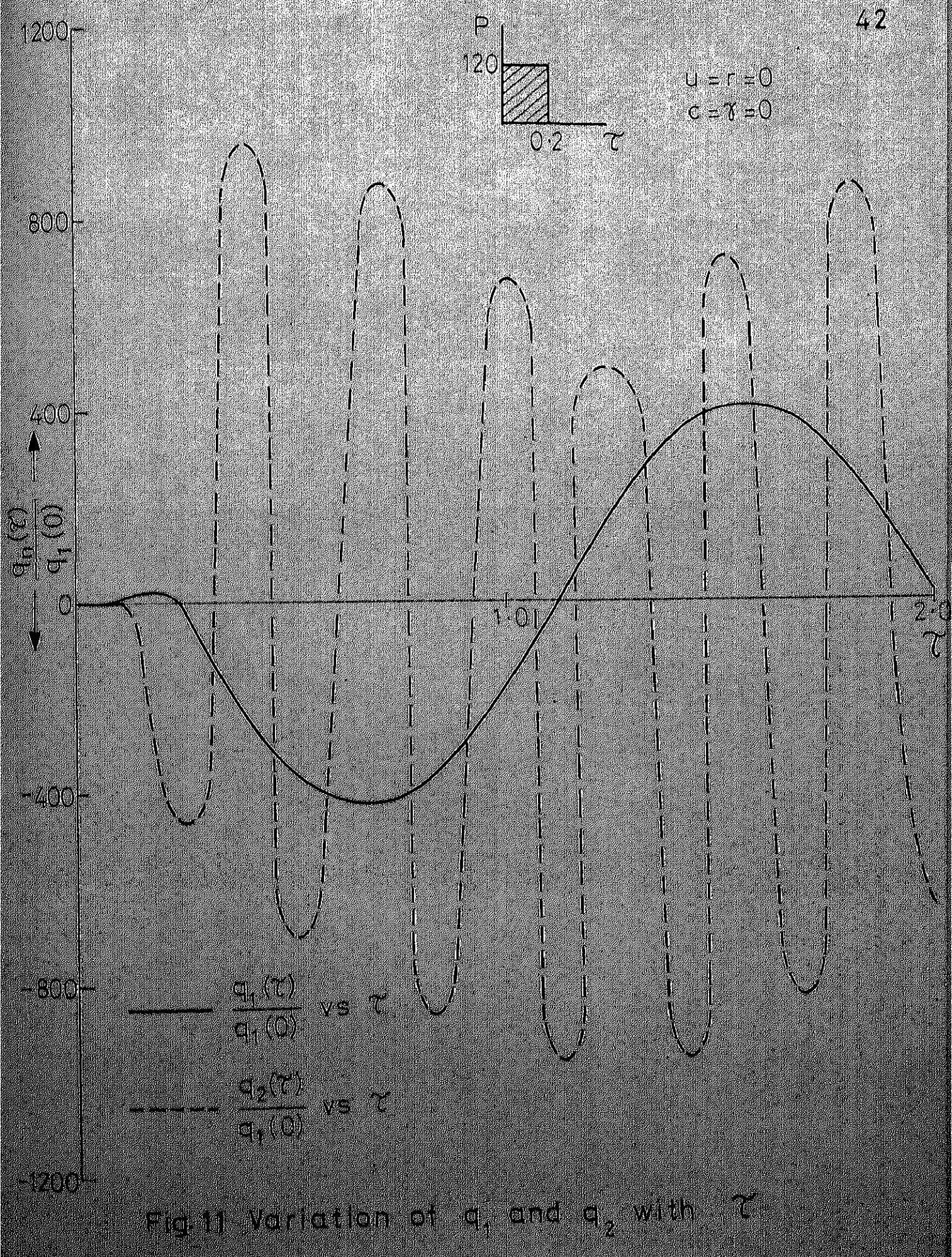


Fig. 11. Variation of  $q_1$  and  $q_2$  with  $\tau$ .

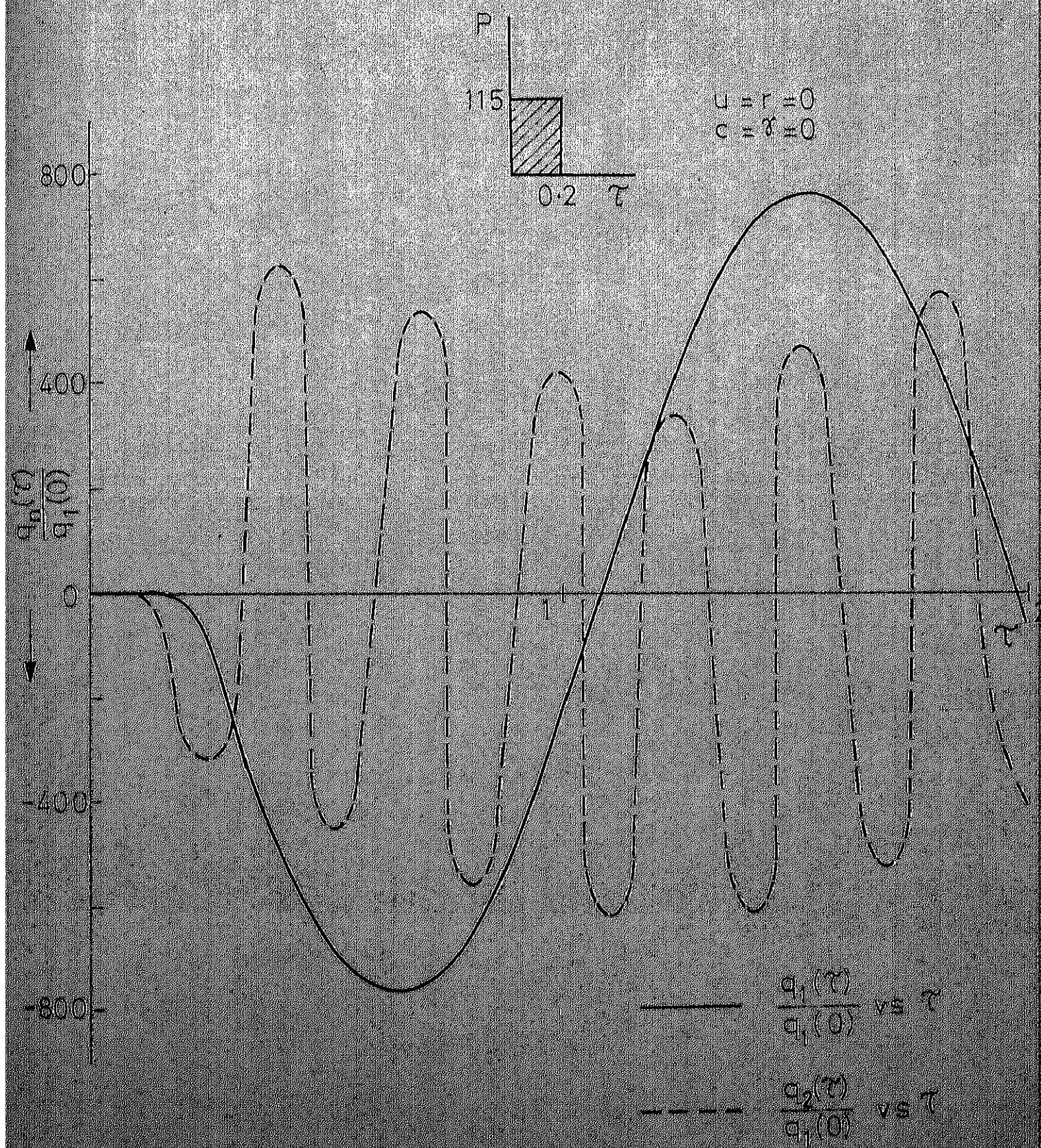


Fig. 12 Variation of  $q_1$  and  $q_2$  with  $\tau$

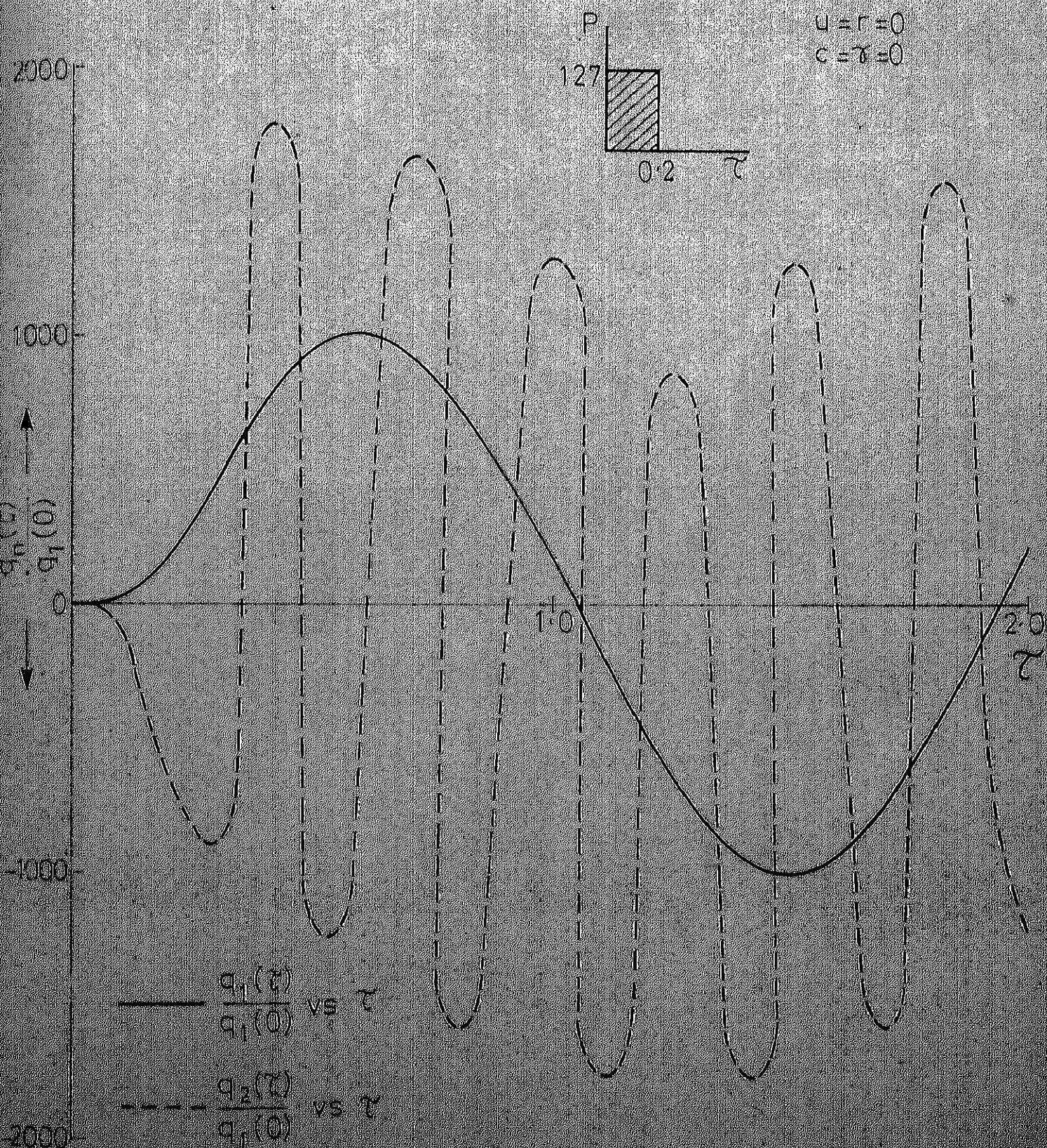


Fig. 1.3 Variation of  $q_1$  and  $q_2$  with  $\tau$

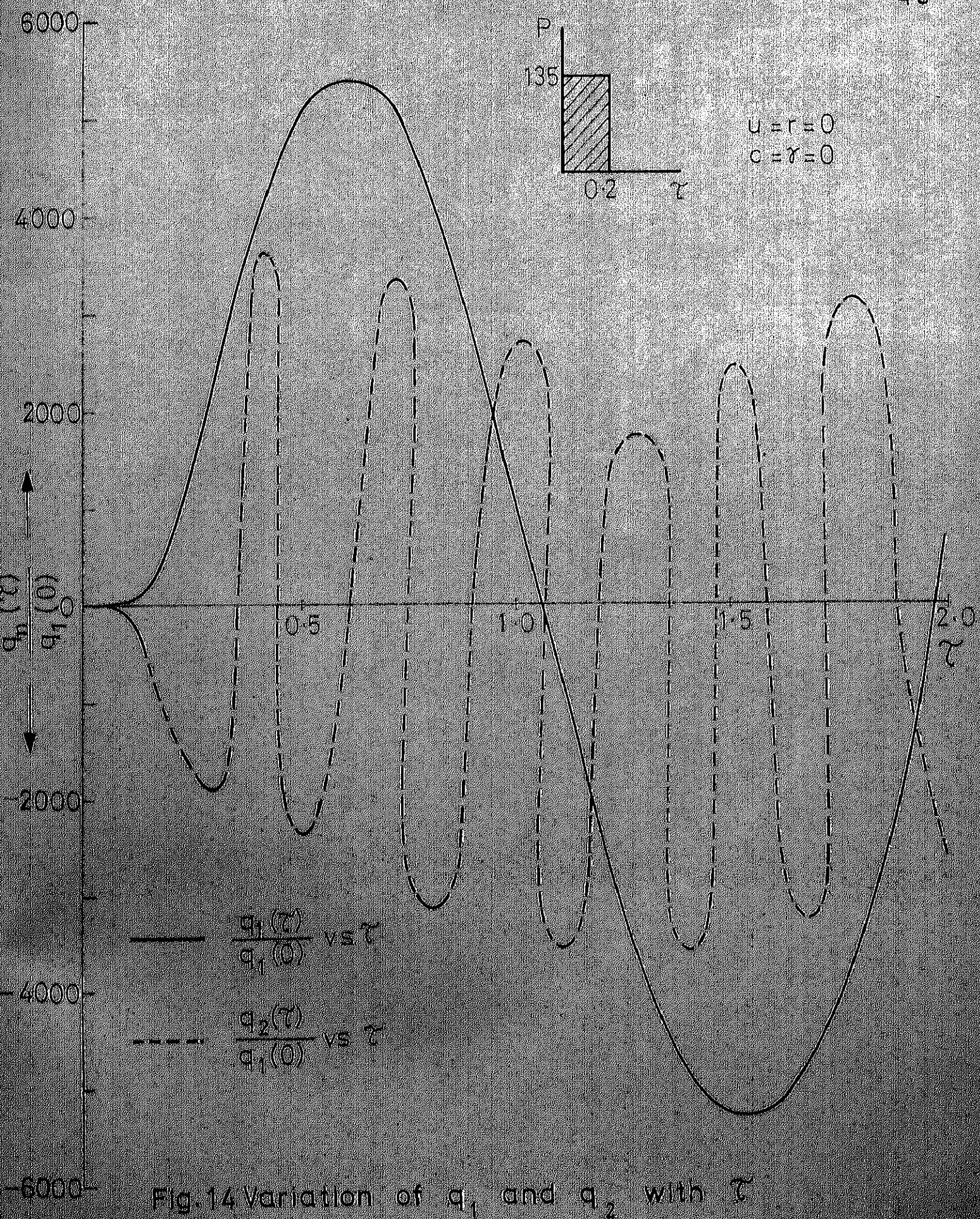


Fig. 14 Variation of  $q_1$  and  $q_2$  with  $\tau$

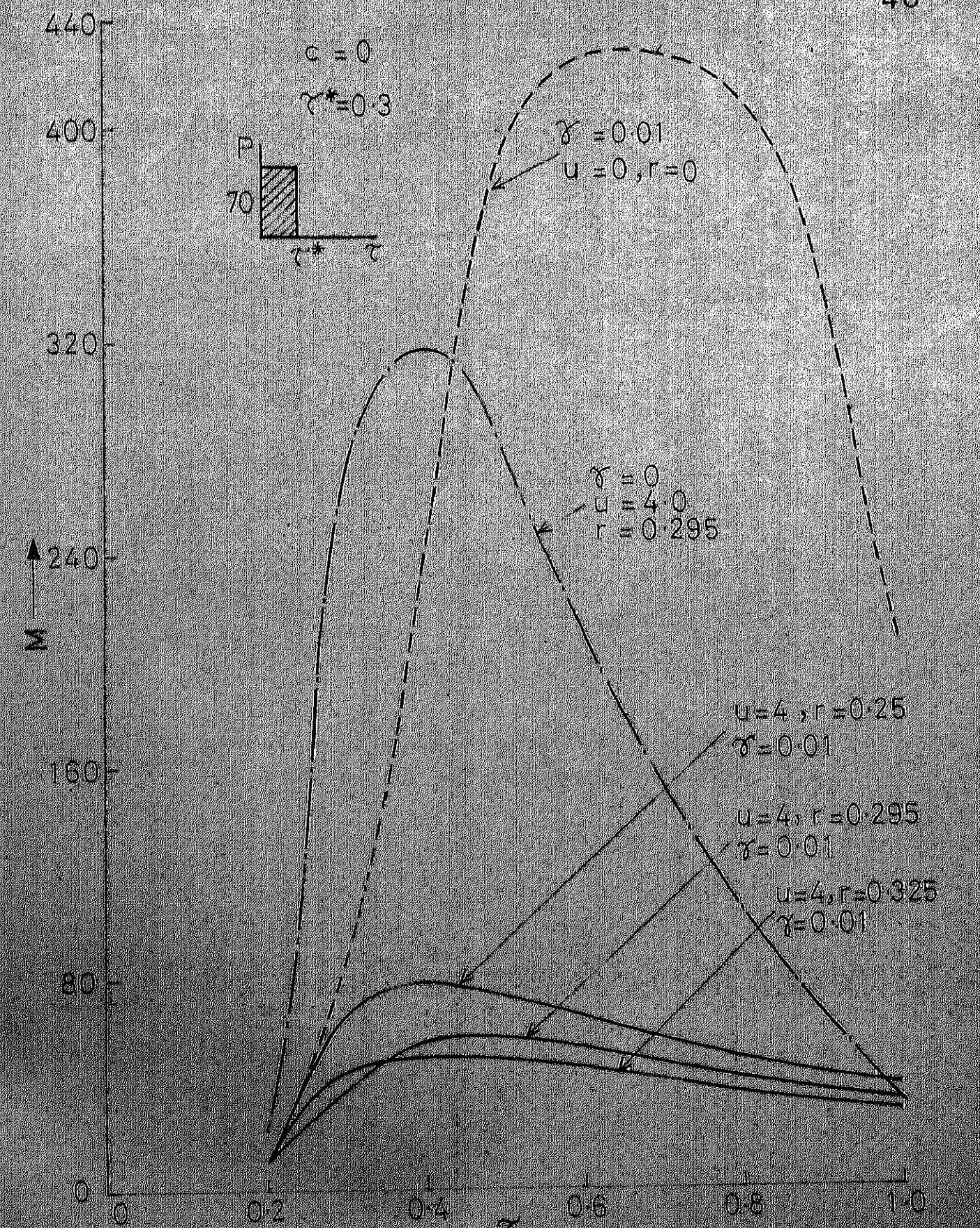


Fig. 15 Variation of magnetization factor with time

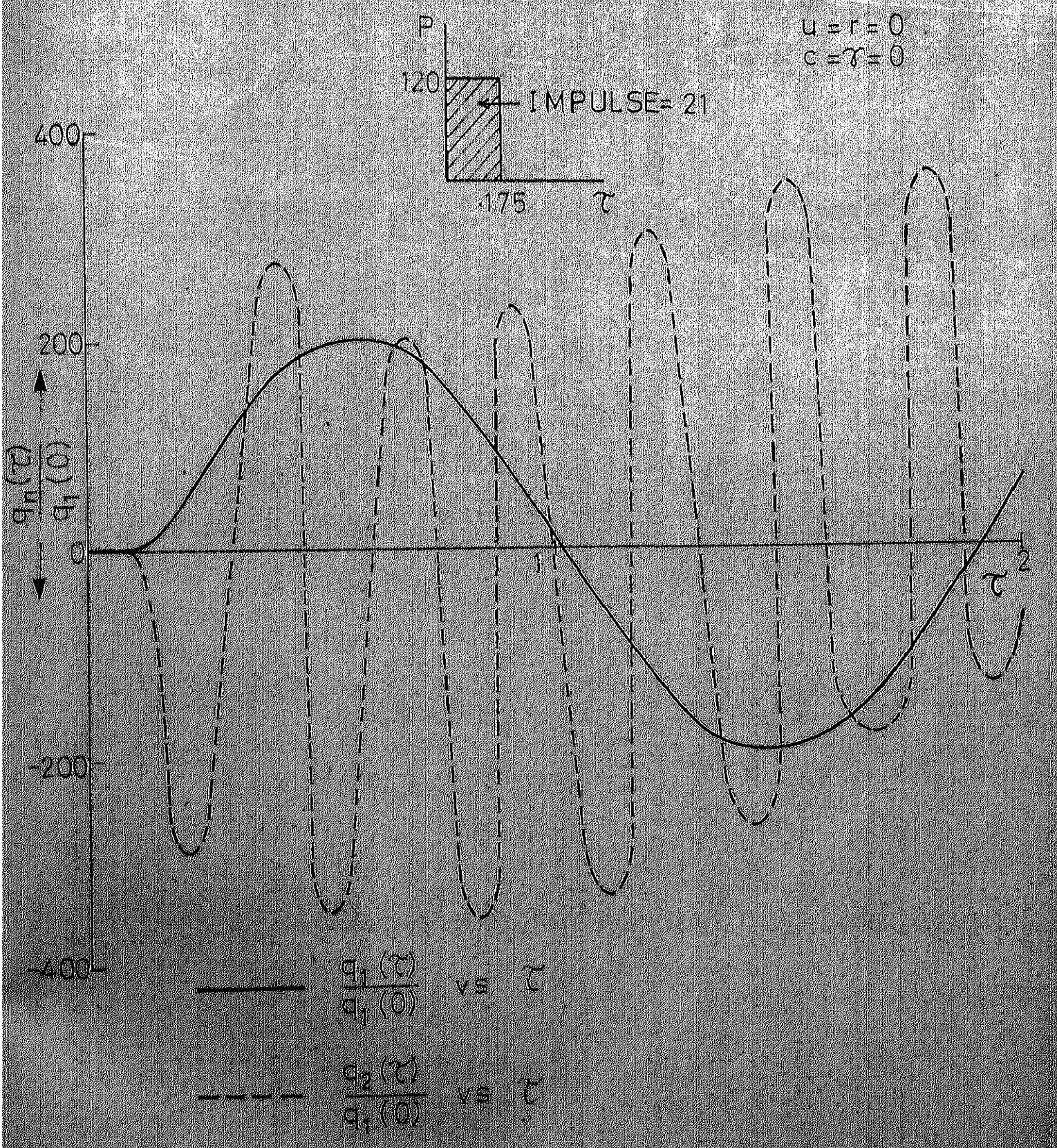


Fig. 16 Variation of  $q_1$  and  $q_2$  with  $\tau$

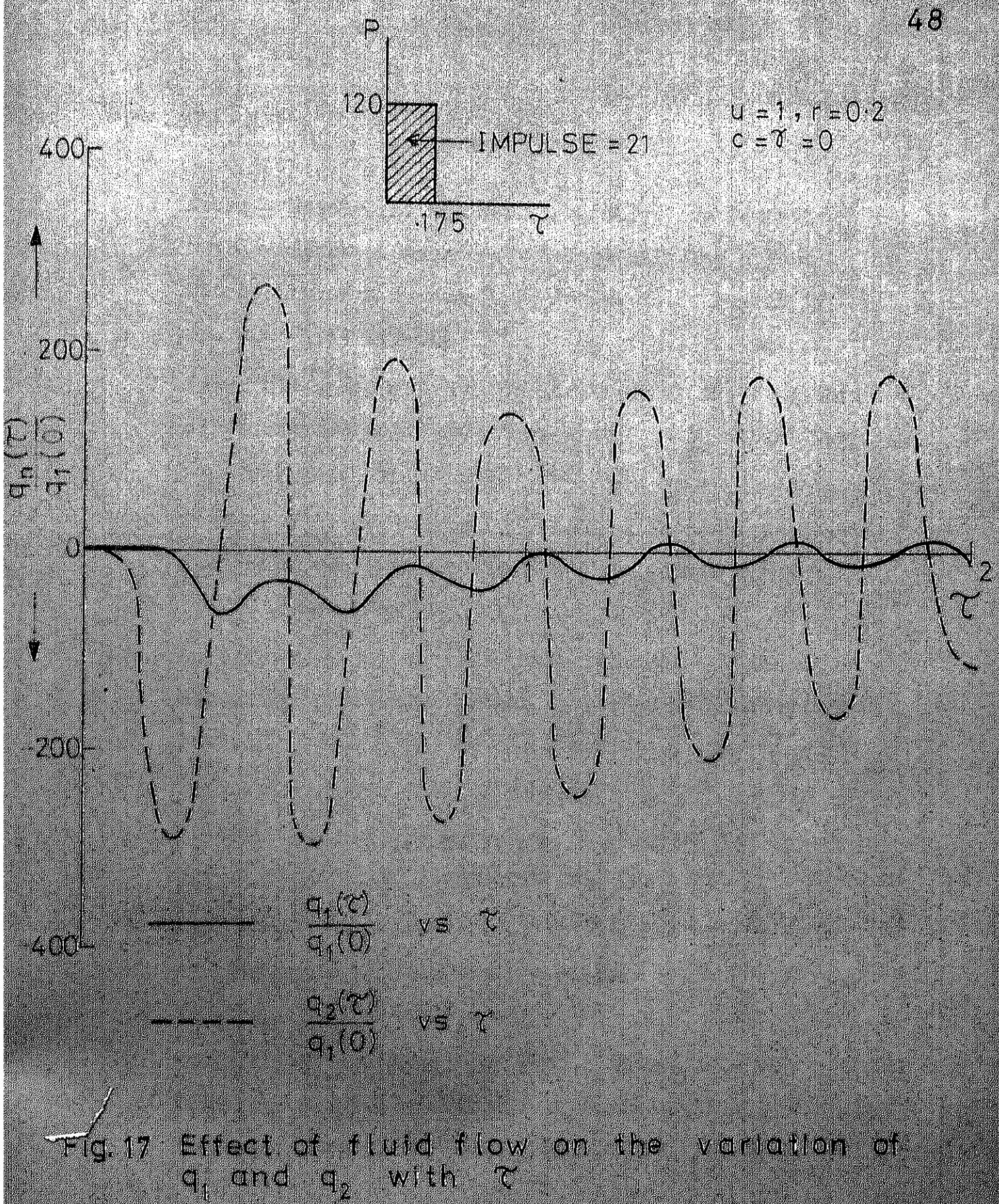


Fig. 17 Effect of fluid flow on the variation of  $q_1$  and  $q_2$  with  $\tau$

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## APPENDIX A

The Values of  $\beta_m$  and  $\alpha_m$  Involved in Equation (3 2)

m	$\beta_m$	$\alpha_m$
1	1.8751041	0.7340955
2	4.6940911	1.01846644
3	7.8547474	0.99922450
4	10.9955407	1.000033553
5	14.1371684	0.9999985501
6	17.27875957	
7	20.42035223	
8	23.56194488	
9	26.70353753	
10	29.84513018	

For  $m > 5$

Since,  $\beta_m = (2m - 1) \frac{\pi}{2}$ ,  $\alpha_m \approx 1.0$

## APPENDIX B

The second order coupled linear ordinary differential Eqs. (3.7) are reduced to first order coupled linear ordinary differential equations for performing the solution by Haming's method

The second order differential equations are

$$\begin{aligned} q_m + \gamma \beta_m^{\frac{1}{4}} \dot{q}_m + \{ P(\tau) + u^2 \} \sum_n \phi_{mn} q_n + 2 u r \sum_n \psi_{mn} q_m \\ + 2c q_m + \beta_m^{\frac{1}{4}} \ddot{q}_m = 0, \quad m = 1, 2, 3, 4, 5 \end{aligned} \quad (B - 1)$$

For  $m = 1$ , Eq. (B-1) can be written in expanded form as

$$\begin{aligned} q_1 + \gamma \beta_1^{\frac{1}{4}} \dot{q}_1 + \{ P(\tau) + u^2 \} \{ \phi_{11} q_1 + \phi_{12} q_2 + \phi_{13} q_3 \\ + \phi_{14} q_4 + \phi_{15} q_5 \} + 2 u r \{ \psi_{11} q_1 + \psi_{12} q_2 + \psi_{13} \dot{q}_3 \\ + \psi_{14} \dot{q}_4 + \psi_{15} \dot{q}_5 \} + 2c \dot{q}_1 + \beta_1^{\frac{1}{4}} \ddot{q}_1 = 0 \end{aligned}$$

Similarly other equations can be written in expanded form.

$$\begin{aligned} \text{Let } q_1 = y_1, \quad q_2 = y_3, \quad q_3 = y_5, \quad q_4 = y_7, \quad q_5 = y_9 \\ \dot{q}_1 = y_2, \quad q_2 = y_4, \quad \dot{q}_3 = y_6, \quad \dot{q}_4 = y_8, \quad \dot{q}_5 = y_{10} \end{aligned}$$

then Eq. (B - 1) can be written in the following form for different values of  $m$ ,

$$\dot{q}_1 = \frac{d y_1}{d \tau} = f_1 (\tau, y_1, \dots, y_m) = y_2 \quad (B - 2)$$

$$\begin{aligned}
 q_1 &= \frac{d y_2}{d \tau} = f_2 (\tau, y_1, \dots, y_m) \\
 &= -\gamma \beta_1^4 y_2 - \{ P(\tau) + u^2 \} \{ \phi_{11} y_1 + \phi_{12} y_3 \\
 &\quad + \phi_{13} y_5 + \phi_{14} y_7 + \phi_{15} y_9 \} - 2 u r \{ \psi_{11} y_2 \\
 &\quad + \psi_{12} y_4 + \psi_{13} y_6 + \psi_{14} y_8 + \psi_{15} y_{10} \} - 2c y_2 - \beta_1^4 y_1
 \end{aligned}$$

(B - 3)

$$q_2 = \frac{d y_3}{d \tau} = f_3 (\tau, y_1, \dots, y_m) = y_4 \quad (B - 4)$$

$$\begin{aligned}
 q_2 &= \frac{d y_4}{d \tau} = f_4 (\tau, y_1, \dots, y_m) \\
 &= \gamma \beta_2^4 y_4 - \{ P(\tau) + u^2 \} \{ \phi_{21} y_1 + \phi_{22} y_3 + \phi_{23} y_5 \\
 &\quad + \phi_{24} y_7 + \phi_{25} y_9 \} - 2 u r \{ \psi_{21} y_2 + \psi_{22} y_4 \\
 &\quad + \psi_{23} y_6 + \psi_{24} y_8 + \psi_{25} y_{10} \} - 2c y_4 - \beta_2^4 y_3
 \end{aligned}$$

(B - 5)

$$q_3 = \frac{d y_5}{d \tau} = f_5 (\tau, y_1, \dots, y_m) = y_6 \quad (B - 6)$$

$$\begin{aligned}
 q_3 &= \frac{d y_6}{d \tau} = f_6 (\tau, y_1, \dots, y_m) \\
 &= -\gamma \beta_3^4 y_6 - \{ P(\tau) + u^2 \} \{ \phi_{31} y_1 + \phi_{32} y_3 \\
 &\quad + \phi_{33} y_5 + \phi_{34} y_7 + \phi_{35} y_9 \} - 2 u r \{ \psi_{31} y_2 + \psi_{32} y_4 \\
 &\quad + \psi_{33} y_6 + \psi_{34} y_8 + \psi_{35} y_{10} \} - 2c y_6 - \beta_3^4 y_5
 \end{aligned}$$

(B - 7)

$$\dot{q}_4 = \frac{d y_7}{d \tau} = f_7 (\tau, y_1, \dots, y_m) = y_8 \quad (B - 8)$$

$$\begin{aligned}
 q_4 &= \frac{d y_8}{d \tau} = f_8 (\tau, y_1, \dots, y_m) \\
 &= -\gamma \beta_4^4 y_8 - \{ P(\tau) + u^2 \} \{ \phi_{41} y_1 + \phi_{42} y_3 \\
 &\quad + \phi_{43} y_5 + \phi_{44} y_7 + \phi_{45} y_9 \} - 2 u r \psi_{41} y_2 \\
 &\quad + \psi_{42} y_4 + \psi_{43} y_6 + \psi_{44} y_8 + \psi_{45} y_{10} \} - 2c y_8 \beta_4^4 y_7
 \end{aligned}$$

$$\dot{q}_5 = \frac{d y_9}{d \tau} = f_9 (\tau, y_1, \dots, y_m) = y_{10} \quad (B-9)$$

$$\begin{aligned}
 \dot{q}_5 &= \frac{d y_9}{d \tau} = f_{10} (\tau, y_1, \dots, y_m) = -\gamma \beta_5^4 y_{10} \\
 &\quad - \{ P(\tau) + u^2 \} \{ \phi_{51} y_1 + \phi_{52} y_3 + \phi_{53} y_5 + \phi_{54} y_7 \\
 &\quad + \phi_{55} y_9 \} - 2 u r \{ \psi_{51} y_2 + \psi_{52} y_4 + \psi_{53} y_6 \\
 &\quad + \psi_{54} y_8 + \psi_{55} y_{10} \} - 2c y_{10} - \gamma \beta_5^4 y_9 \quad (B-11)
 \end{aligned}$$

The above ten simultaneous first order linear differential Eqs (B-2 to B-11) are solved by Haming's method (Predictor - corrector method)

## APPENDIX C

In case of cantilever beam the empirical formulae [19] for determining the integrals  $\phi_{mn}$  and  $\psi_{mn}$  are given below

$$\begin{aligned}\phi_{mn} &= \int_0^1 \frac{d^2 \phi_m}{d\xi^2} \phi_n(\xi) d\xi = \beta_m \alpha_m (2 - \beta_m \alpha_m), \quad m = n \\ &= \frac{4 (\beta_m \alpha_m - \beta_n \alpha_n)}{(-1)^{m+n} - \left(\frac{\beta_n}{\beta_m}\right)^2}, \quad m \neq n\end{aligned}$$

$$\psi_{mn} = \int_0^1 \frac{d \phi_m}{d\xi} \phi_n(\xi) d\xi = \frac{4}{\left(\frac{\beta_n}{\beta_m}\right)^2 + (-1)^{m+n}}$$

The integrals are computed numerically by Guassian-Quadrature 15 point formula. The values of these integrals were found approximately same by numerical integration as well as by emperical formulae.

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